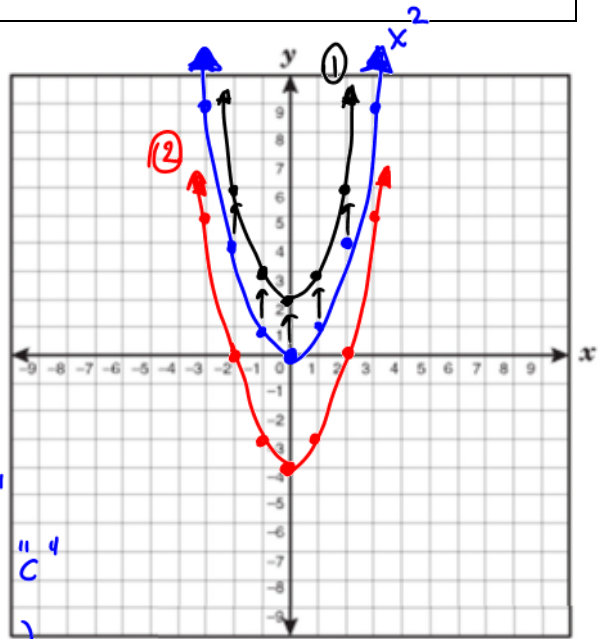


In this activity you will discover the graphical connection between  $y = f(x)$  and the functions of the form:

- $y = f(x) + c$ ,  $c$  is a constant
- $y = f(x - d)$ ,  $d$  is a constant
- $y = af(x)$ ,  $a$  is a constant and  $a > 0$
- $y = f(kx)$ ,  $k$  is a constant and  $k > 0$
- $y = -f(x)$
- $y = f(-x)$

**Investigation: Type  $y = f(x) + c$**

- Sketch the graph of  $f(x) = x^2$
  - On the same set of axes sketch the graphs of
    - ① •  $y = x^2 + 2$
    - ② •  $y = x^2 - 3$



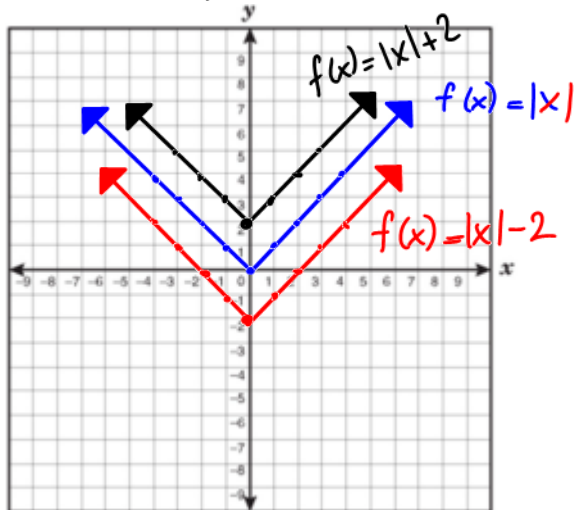
c. If  $y = f(x)$  is transformed to  $y = f(x) + c$ , where  $c$  is a constant, describe the transformation:

- if  $c > 0$ , then graph shifts up as much as "c"
- if  $c < 0$ , then graph shifts down as much as "c"
- Any point  $(x, y)$  under this transformation becomes  $(x, y+c)$

*Notice that only "y" changes, b/c it's a vertical shift*

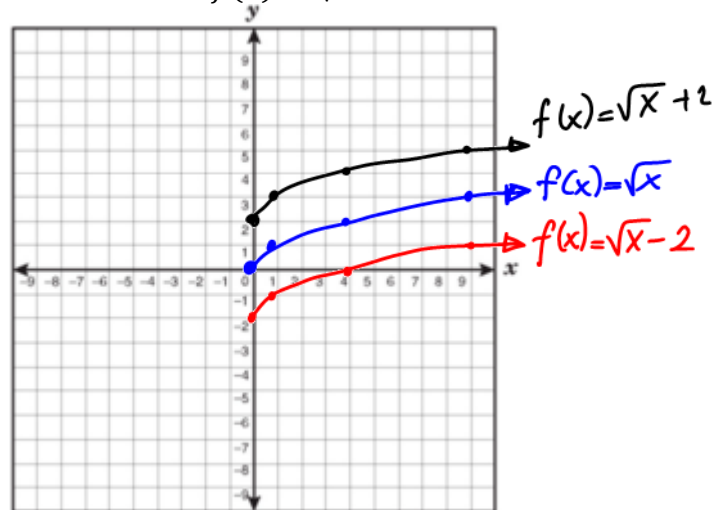
2. Using the pattern you discovered in 1c, sketch  $y = f(x)$ ,  $y = f(x) + 2$ , and  $y = f(x) - 2$  on the same set of axes for each of the following functions:

a.  $f(x) = |x|$



$$f(x) = |x| + 2$$

b.  $f(x) = \sqrt{x}$



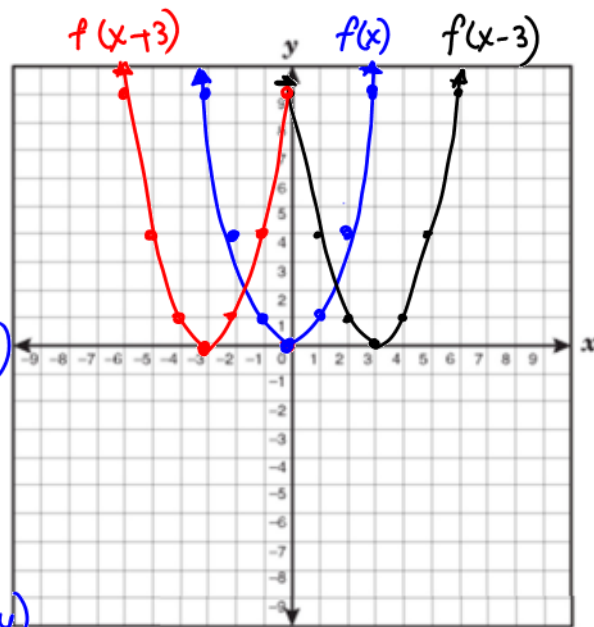
**Investigation: Type  $y = f(x - d)$**

1. a. Sketch the graph of  $f(x) = x^2$
- b. On the same set of axes sketch the graphs of
  - $y = (x - 3)^2 \rightarrow$  vertex  $(3, 0) \rightarrow d = +3$
  - $y = (x + 3)^2 \rightarrow$  vertex  $(-3, 0) \rightarrow d = -3$

Recall  $y = a(x-h)^2 + k$  Vertex is  $(h, k)$   
when "+3" subbed  $(x-3)$ , "-3" subbed  $(x+3)$

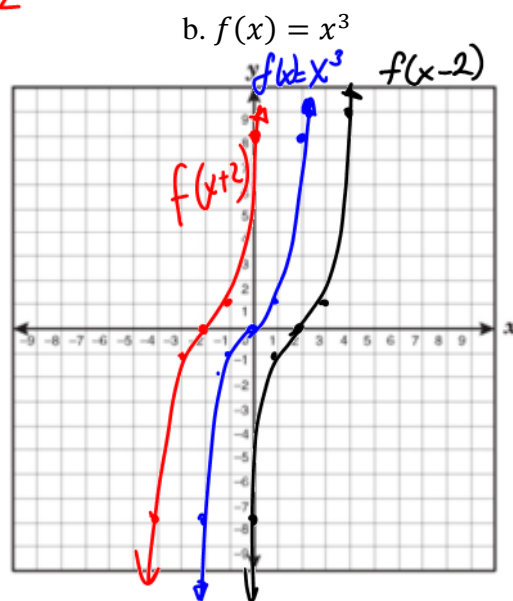
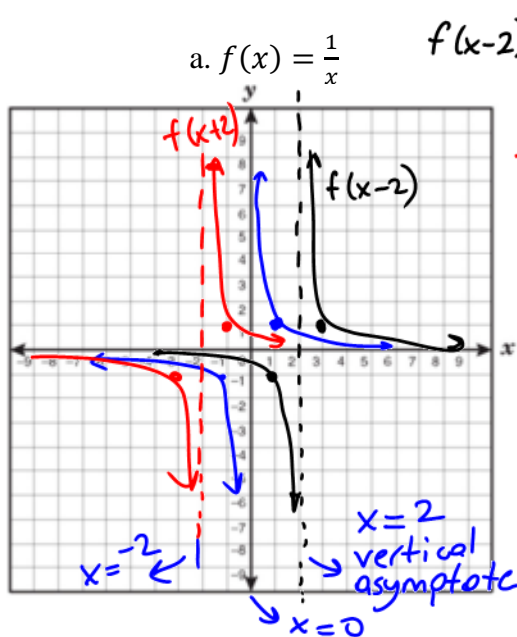
c. If  $y = f(x)$  is transformed to  $y = f(x - d)$ , where  $d$  is a constant, describe the transformation:

- if  $d > 0$ , then graph shifts right " $d$ " units
- if  $d < 0$ , then graph shifts left " $d$ " units
- Any point  $(x, y)$  under this transformation becomes  $(x+d, y)$



This is a horizontal shift; therefore only "x" coordinate changes.

2. Using the pattern you discovered in 1c, sketch  $y = f(x)$ ,  $y = f(x - 2)$ , and  $y = f(x + 2)$  on the same set of axes for each of the following functions:



**COMMUNICATION**

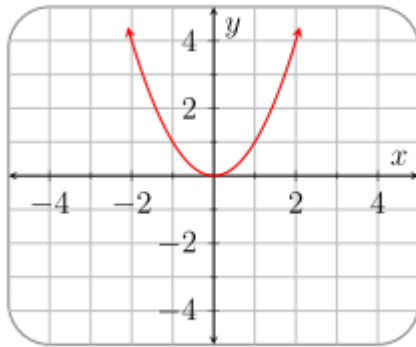
When a function is translated, we use key words such as translate (shift), units, up or down.  
For example,  $y = (x - 2)^2 + 1$  the parent function has been shifted 2 units ~~right~~ and 1 unit up.

**Try.** State the transformations for:  $y = (x + 3)^2 - 2$

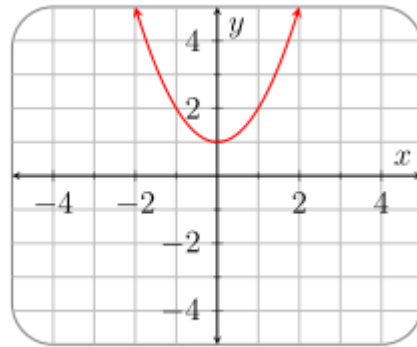
*The parent function has been shifted 3 units left and 2 units down*

Horizontal & Vertical Translation Practice

1. Each of the following graphs show a shift of the function  $f$  that has formula  $f(x) = x^2$ . Describe the shifts involved to obtain the function  $g$ .

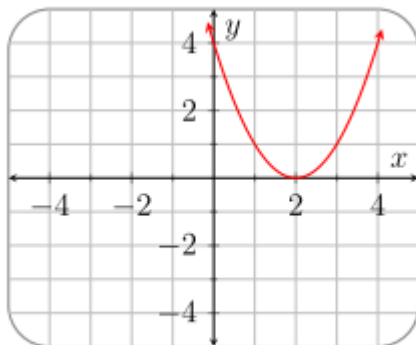


(a)  $f(x) = x^2$



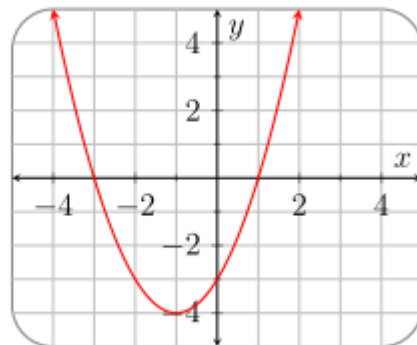
(b)  $g(x) = x^2 + 1$

shifted 1 unit up



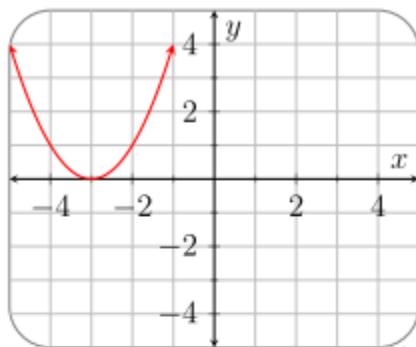
(c)  $g(x) = (x - 2)^2$

shifted 2 units right

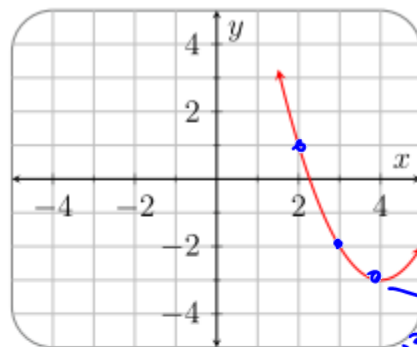


(d)  $g(x) = (x + 1)^2 - 4$

shifted 1 unit left and 4 units down.

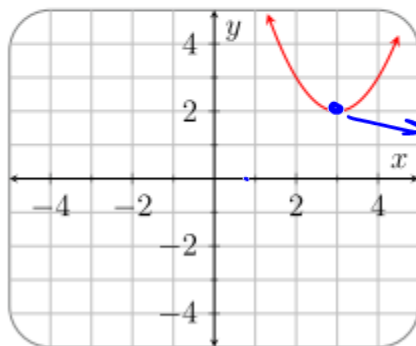


(e)  $g(x) = (x + 3)^2$



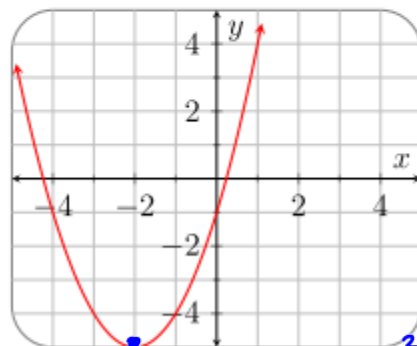
(f)  $g(x) = (x - 4)^2 - 3$

vertex (4, -3)



(g)  $g(x) = (x - 3)^2 + 2$

(3, 2)



(h)  $g(x) = (x + 2)^2 - 4$

(-2, -4)

Figure 1: Shifts of  $f(x) = x^2$ .

2. Consider the graph of  $f$  given in **Figures 2 to 7**. For the shifts given underneath the figures, **state** if they represent a horizontal or a vertical shift, and then sketch this shift on the axis provided.

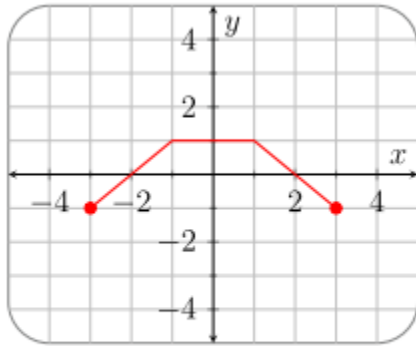


Figure 2:  $f(x)$

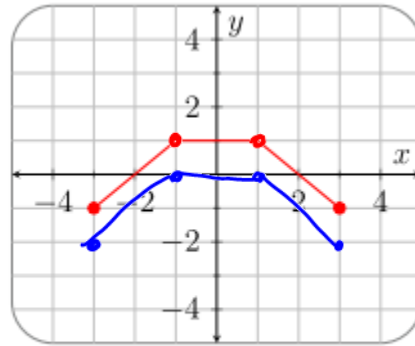
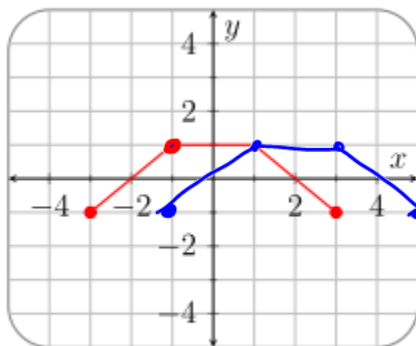
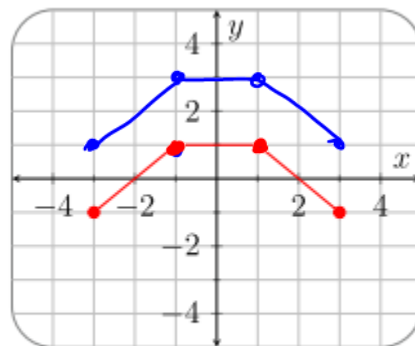


Figure 3:  $f(x) - 1$  vertical shift 1 down



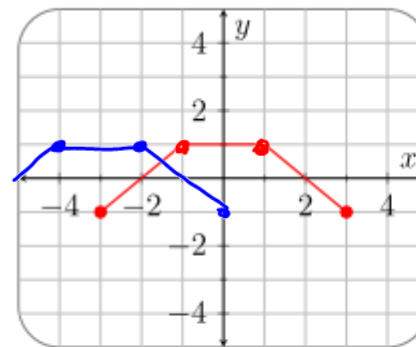
horizontal  
2 right

Figure 4:  $f(x - 2)$



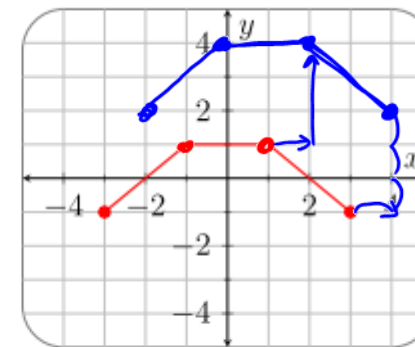
vertical  
2 up

Figure 5:  $f(x) + 2$



horizontal  
3 left

Figure 6:  $f(x + 3)$



1 right  
3 up

Figure 7:  $f(x - 1) + 3$

**SUMMARY**

In general, if  $f$  is a function and  $c$  is constant, then the graph of

$y = f(x) + c$  is the graph of  $y = f(x)$  shifted  
 - **vertically** upwards if  $c > 0$   
 - **vertically** downwards if  $c < 0$ .  
 This is called an **OUTSIDE CHANGE**

$y = f(x - h)$  is the graph of  $y = f(x)$  shifted  
 - **horizontally** to the left if  $h > 0$   
 - **horizontally** to the right if  $h < 0$   
 This is called an **INSIDE CHANGE**