DECOMPOSITION METHOD

Factor $2x^2 + x - 6$

$3 \quad {}^-4$

The two numbers that add to +1 and multiply to -12 are . . .

$$= 2x^2 + 3x - 4x - 6$$
$$= x(2x+3) - 2(2x+3)$$
$$= (2x+3)(x-2)$$

Split the middle term using the two numbers you found, mx and nx.

Factor these 4 terms by grouping. (i.e. common factor the first two terms, common factor the last two terms - Remember: the two binomials in the brackets should always be the same).

Common factor the binomials in the brackets.

AUSTRALIAN METHOD

Example 1: Factor $2x^2 + x - 6$

$3 \quad {}^-4$

The two numbers that add to +1 and multiply to -12 are . . .

$$= \frac{(2x+3)(2x-4)}{2}$$

$$= \frac{(2x+3)(2)(x-2)}{2}$$

$\rightarrow (2x+3)(x-2)$

Write the following terms in this form:
$$\frac{(ax+m)(ax+n)}{a}.$$
Common factor the two sets of brackets.
Reduce the fraction.

Example: Factor $2x^2 + 11x + 5$	*Example*: Factor $6x^2 - 7x + 2$
m and n must . . . + to __11__, × to __10__	m and n must . . . + to __-7__, × to __12__
m and n are $^+1$, $^+10$	m and n are $^-3$, $^-4$
$2x^2 + 11x + 5$	**Aus** $6x^2 - 7x + 2$ **DEC**
$= 2x^2 + x + 10x + 5$	$= \frac{(6x-3)(6x-4)}{6}$ $6x^2 - 3x - 4x + 2$
$= x(2x+1) + 5(2x+1)$ *decomp.*	$= \frac{(3)(2x-1)(2)(3x-2)}{6}$ $= 3x(2x-1) - 2(2x-1)$
$= (2x+1)(x+5)$	$= (2x-1)(3x-2)$ $= (2x-1)(3x-2)$
Example 4: Factor $4x^2 - 4x - 3$	*Example 5*: Factor $8x^2 + 2x - 3$
m and n must . . . + to __-4__, × to __-12__	m and n must . . . + to __+2__, × to __-24__
m and n are $^+2$, $^-6$	m and n are $^-4$, $^+6$
DEC $4x^2 - 4x - 3$ **or** **Aus**	$8x^2 + 2x - 3$ **Aus**
$4x^2 - 6x + 2x - 3$ $= \frac{(4x-6)(4x+2)}{4}$	$8x^2 - 4x + 6x - 3$ $= \frac{(8x-4)(8x+6)}{8}$
$= 2x(2x-3) + (2x-3)$ $= \frac{2(2x-3)(2)(2x+1)}{4}$	$= 4x(2x-1) + 3(2x-1)$ $= \frac{4(2x-1)(2)(4x+3)}{8}$
$= (2x-3)(2x+1)$ $= (2x-3)(2x+1)$	$= (2x-1)(4x+3)$ $= (2x-1)(4x+3)$

Example 6: Factor $4x^2 - 27x + 18$

m and n must . . . + to __-27__ , × to __72__

m and n are __-3__ , __-24__

$4x^2 - 27x + 18$

$= 4x^2 - 3x - 24x + 18$

$= x(4x-3) - 6(4x-3)$

$= (4x-3)(x-6)$

$= \dfrac{(4x-3)(4x-24)}{4}$

$= \dfrac{(4x-3)(4)(x-6)}{4}$

$= (4x-3)(x-6)$

Example 7: Factor $9x^2 - 6x - 8$

m and n must . . . + to __-6__ , × to __-72__

m and n are __$+6$__ , __-12__

$9x^2 - 6x - 8$

$= 9x^2 - 12x + 6x - 8$

$= 3x(3x-4) + 2(3x-4)$

$= (3x-4)(3x+2)$

$\dfrac{(9x-12)(9x+6)}{9}$

$= \dfrac{3(3x-4)(3)(3x+2)}{9}$

$= (3x-4)(3x+2)$

Example 8: Factor $6x^2 - 7x - 3$

m and n must . . . + to __-7__ , × to __-18__

m and n are __$+2$__ , __-9__

$6x^2 - 7x - 3$

$6x^2 - 9x + 2x - 3$

$= 3x(2x-3) + (2x-3)$

$= (2x-3)(3x+1)$

$\dfrac{(6x-9)(6x+2)}{6}$

$= \dfrac{3(2x-3)(2)(3x+1)}{6}$

$= (2x-3)(3x+1)$

Example 9: Factor $4x^2 - 13x - 12$

m and n must . . . + to __-13__ , × to __-48__

m and n are __3__ , __-16__

$4x^2 - 13x - 12$

$4x^2 - 16x + 3x - 12$

$= 4x(x-4) + 3(x-4)$

$= (x-4)(4x+3)$

$\dfrac{(4x-16)(4x+3)}{4}$

$= \dfrac{4(x-4)(4x+3)}{4}$

$= (x-4)(4x+3)$

Example 10: Factor $5x^2 + 13x - 6$

m and n must . . . + to __13__ , × to __-30__

m and n are __-2__ , __15__

$5x^2 + 13x - 6$

$5x^2 - 2x + 15x - 6$

$= x(5x-2) + 3(5x-2)$

$= (5x-2)(x+3)$

$= \dfrac{(5x-2)(5x+15)}{5}$

$= \dfrac{(5x-2)(5)(x+3)}{5}$

$= (5x-2)(x+3)$

Example 11: Factor $8x^2 - 14x + 3$

m and n must . . . + to __-14__ , × to __24__

m and n are __-2__ , __-12__

$8x^2 - 14x + 3$

$8x^2 - 12x - 2x + 3$

$= 4x(2x-3) - 1(2x-3)$

$= (2x-3)(4x-1)$

$\dfrac{(8x-12)(8x-2)}{8}$

$= \dfrac{4(2x-3)(2)(4x-1)}{8}$

$= (2x-3)(4x-1)$

Tricky Practice

-15

$3, -5$ 1) $3p^2 - 2p - 5 = 3p^2 + 3p - 5p - 5$

$\qquad = 3p(p+1) - 5(p+1)$

$\qquad = (p+1)(3p-5)$

-18 $-3, -6$

2) $2n^2 + 3n - 9$

$\quad = 2n^2 - 3n + 6n - 9$

$\quad = n(2n-3) + 3(2n-3)$

$\quad = (2n-3)(n+3)$

$(7a + 4)(a + 7)$

12

3) $3n^2 - 8n + 4$

$= 3n^2 - 2n - 6n + 4$

$= n(3n-2) - 2(3n-2)$

$= (3n-2)(n-2)$

4) $5n^2 + 19n + 12$

M	A	N
60	19	4,15

$= 5n^2 + 4n + 15n + 12$

$= n(5n+4) + 3(5n+4)$

$= (5n+4)(n+3)$

$-(2a + 5)(3a + 5)$

$4(b + 5)(4b - 5)$

$3(3k + 1)(k + 7)$

5) $2v^2 + 11v + 5$

M	A	N
10	11	1,10

$= \dfrac{(2v+1)(2v+10)}{2}$

$= \dfrac{(2v+1)(2)(v+5)}{2} \Rightarrow (2v+1)(v+5)$

6) $2n^2 + 5n + 2$

M	A	N
4	5	1,4

$= \dfrac{(2n+1)(2n+4)}{2}$

$= \dfrac{(2n+1)(2)(n+2)}{2} = (2n+1)(n+2)$

$(n - 4)(4n - 1)$

$(n - 5)(4n + 5)$

$(2n + 1)(n + 2)$

7) $7a^2 + 53a + 28$

M	A	N
196	53	4,49

$= 7a^2 + 4a + 49a + 28$

$= a(7a+4) + 7(7a+4)$

$= (7a+4)(a+7)$

8) $9k^2 + 66k + 21$

M	A	N
189	66	3, 63

$= 9k^2 + 3k + 63k + 21$

$= 3k(3k+1) + 21(3k+1)$

$= (3k+1)(3k+21)$

$= (3k+1)(3)(k+7)$

$(2n + 3)(3n - 2)$

$(2n - 3)(n + 3)$

9) $15n^2 - 27n - 6$

M	A	N
-90	-27	3, -30

$= 15n^2 - 30n + 3n - 6$

$= 15n(n-2) + 3(n-2)$

$= (n-2)(15n+3)$

$= (n-2)(3)(5n+1)$

10) $5x^2 - 18x + 9$

M	A	N
45	-18	3, -15

$= 5x^2 - 15x - 3x + 9$

$= 5x(x-3) - 3(x-3)$

$= (x-3)(5x-3)$

$(3n - 2)(n - 2)$

$(5n + 4)(n + 3)$

11) $4n^2 - 15n - 25$

M	A	N
-100	-15	5, -20

$= 4n^2 - 20n + 5n - 25$

$= 4n(n-5) + 5(n-5)$

$= (n-5)(4n+5)$

12) $4x^2 - 35x + 49$

M	A	N
196	-35	-7, -28

$= 4x^2 - 28x - 7x + 49$

$= 4x(x-7) - 7(x-7)$

$= (x-7)(4x-7)$

$3(5n + 1)(n - 2)$

$(3p - 5)(p + 1)$

$(2v + 1)(v + 5)$

15) $6x^2 + 37x + 6$

M	A	N
36	37	1, 36

$= 6x^2 + x + 36x + 6$

$= x(6x+1) + 6(6x+1)$

$= (6x+1)(x+6)$

16) $-6a^2 - 25a - 25$

M	A	N
150	-25	-10, 15

$= -6a^2 - 15a - 10a - 25$

$= -3a(2a+5) - 5(2a+5)$

$= (2a+5)(-3a-5)$

$= (2a+5)(-1)(3a+5)$

$(x - 7)(4x - 7)$

$(x + 6)(6x + 1)$

$(3x - 7)(2x + 7)$

17) $6n^2 + 5n - 6$

M	A	N
-36	5	-4, +9

$= 6n^2 - 4n + 9n - 6$

$= 2n(3n-2) + 3(3n-2)$

$= (3n-2)(2n+3)$

18) $16b^2 + 60b - 100$

M	A	N
-100	15	-5, 20

$4(4b^2 + 15b - 25)$

$= 4(4b^2 - 5b + 20b - 25)$

$= 4[b(4b-5) + 5(4b-5)]$

$= 4(4b-5)(b+5)$

$(5x - 3)(x - 3)$