

The Present Value of an Annuity

The *present value* of an annuity is the *principal* that must be invested **TODAY** to provide regular payments in the future.

The **PRESENT VALUE** of an ordinary simple annuity is given by the formula $PV = \frac{R[1-(1+i)^{-n}]}{i}$, where

$PV =$ Amount invested now

$i =$ interest rate in decimal per compounding period

$R =$ Payments per compounding period

$n =$ number of compounding periods.

This formula can only be used when the *payment interval is the same as the compounding period*

Example 1 – Providing for an Annuity

Victor wants to withdraw \$700 at the end of each month for 8 months, starting 1 month from now. His bank account earns 5.4% per year compounded monthly.

a) How much must Victor deposit in his account TODAY to pay for the withdrawals?

Type: compounded monthly

$$PV = \frac{R[1-(1+i)^{-n}]}{i}$$

$PV = ?$ how much do Vic invest now in an account that pays 5.4% per year

$R =$ \$700

$i =$ 5.4% / 1 year $\xrightarrow{\div 12}$ $0.054 \div 12 = 0.0045$

$n =$ 8 months

$$= \frac{700(1-(1+0.0045)^{-8})}{0.0045}$$

$$= \$5488.28$$

\therefore Victor must invest in an account now to be able to take out \$700 for 8 months.

The **INTEREST** of an ordinary simple annuity is given by the formula $I = Rn - PV$, where I is interest

b) How much interest did the annuity earn?

Victor will take out $700 \cdot 8 = \$5600$ in 8 months.

Victor has to invest \$5488 today

$$\begin{aligned} \text{Interest earned} &= 5600 - 5488 \\ &= \$112. \end{aligned}$$

\therefore As Victor withdraws \$700 every month, his original investment starts to decrease. Yet, the remaining value continues to earn interest. In total his investment earned \$112 in 8 months.

Example 2 – Calculating the Amount Needed at Retirement

Azad plans to retire at age 60. He would like to have enough money saved in his retirement account so he can withdraw \$7500 every 3 months for 25 years, starting 3 months after he retires. How much must Azad deposit at retirement at 9% per year compounded quarterly to provide for the annuity?

We need to figure out the lump sum needs to be invested when Azad turns 60 at an investment earns 9% per year comp quarterly so that he can collect \$7500 every 3 mth. for 25 years.

Type: compound quarterly

PV: ?

R: \$7500

i: 9%/year $\xrightarrow{\div 4}$ $0.09 \div 4 = 0.0225$

n: 25 years $\xrightarrow{\times 4}$ 100 periods

$$PV = \frac{R [1 - (1+i)^{-n}]}{i}$$

$$= \frac{7500 [1 - (1 + 0.0225)^{-100}]}{0.0225}$$

$$= \$297313.05$$

\therefore He needs to invest \$297,313.05 the day when he turns 60.

Example 3 – Calculating the Principal Borrowed for a Loan

Valeria plans to buy a used car. She can afford monthly car loan payments of \$300. The car dealer offers Valeria a loan at 6.9% per year compounded monthly, for 3 years. The first payment will be made 1 month from the date she buys the car.

a) How much can Valeria afford to borrow?

Type: compounded monthly

PV: ? (How much is Valeria borrowing)

R: \$300

i: 6.9%/year $\xrightarrow{\div 12}$ $0.069 \div 12 = 0.00575$

n: 3 years $\xrightarrow{\times 12}$ 36 periods

$$PV = \frac{R [1 - (1+i)^{-n}]}{i}$$

$$= \frac{300 (1 - (1 + 0.00575)^{-36})}{0.00575}$$

$$= 9730.34$$

\therefore Valeria borrows \$9730.34

b) How much interest will Valeria pay on the loan?

Valeria will make 36 payments of \$300; thus, $36 \times 300 = \$10800$

She is borrowing \$9730.34

The difference is the interest Valeria will pay on the loan

$$= 10800 - 9730.34$$

$$= \underline{\underline{\$1069.66}}$$

PRACTICE

1. How much do you need to invest now at an ordinary simple annuity so that you can receive payments of \$240 every month for 15 years at 2.25% per year compounded monthly?

Type = Compounded monthly

PV = ?

2) Determine the present value and amount of interest annuity earned of the following ordinary simple annuity: Quarterly payments of \$50 for 4.5 years at 4.8% per year compounded quarterly

3) Mikayla is setting up an income fund for her retirement. She wishes to receive \$2500 every month for the next 20 years, starting 1 month from now. The income fund pays 4% per year compounded monthly. How much must Mikayla deposit now to be able to receive the desired amount every month?