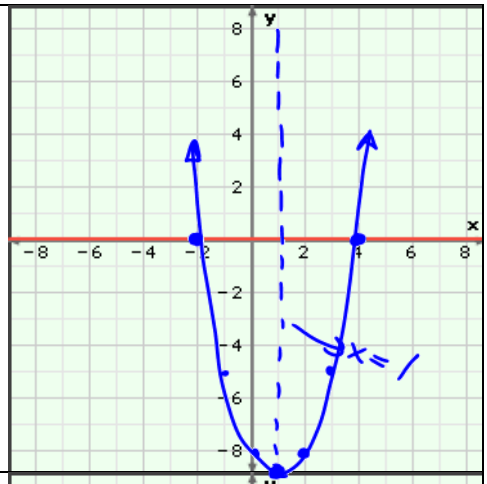
MPM2D1 **Date:** _____

Day 7: Finding Vertex - Averaging Zeros **Chapter 6: Quadratic Equations**

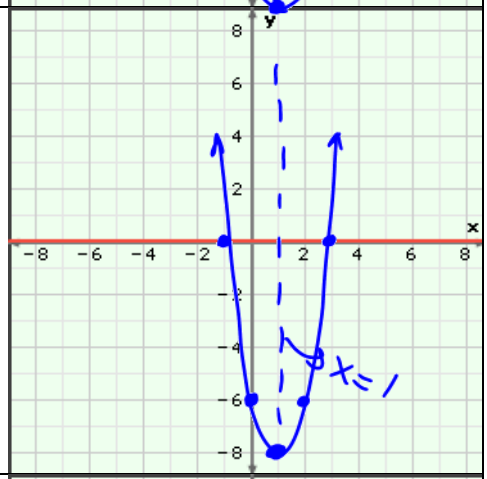
direction, x-intercepts

$y = a(x-r)(x-s)$

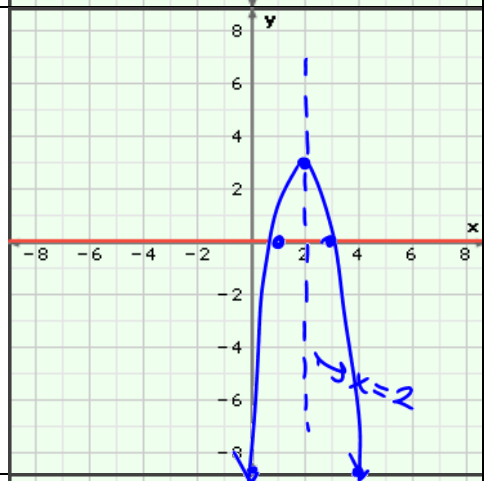
$y = (x+2)(x-4)$	
x-intercepts/zeros: $x+2=0$ $x=-2$ $x-4=0$ $x=4$	Vertex: axis of symmetry is the x coordinate of your vertex $x=\frac{-2+4}{2}=\frac{2}{2}=1$ sub "1" for "x" in the equa. to get the "y" coordinate $y=(1+2)(1-4)$ $=(3)(-3)$ $=-9$ Vertex $(1,-9)$



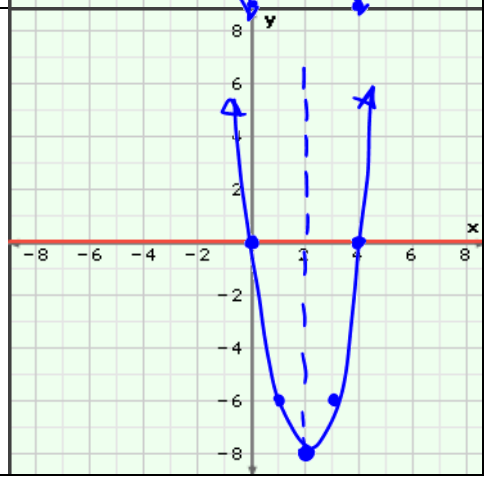
$y = 2(x+1)(x-3)$	
x-intercepts/zeros: $x+1=0$ $x=-1$ $x-3=0$ $x=3$	Vertex: $x=\frac{-1+3}{2}=\frac{2}{2}=1$ $y=2(x+1)(x-3)$ $=2(1+1)(1-3)$ $=2(2)(-2)$ $y=-8$ $V(1,-8)$ Steps = 2, 6, 10



$y = -3(x-3)(x-1)$	
x-intercepts/zeros: $x-3=0$ $x=3$ $x-1=0$ $x=1$	Vertex: $x=\frac{1+3}{2}=\frac{4}{2}=2$ $y=-3(x-3)(x-1)$ $=-3(2-3)(2-1)$ $=-3(-1)(1)$ $=3$ $V(2,3)$ Steps = $-3, -9$



$y = 2x(x-4)$	
x-intercepts/zeros: $2x=0$ $x=0$ $x-4=0$ $x=4$	Vertex: $x=\frac{0+4}{2}=\frac{4}{2}=2$ $y=2x(x-4)$ $=2(2)(2-4)$ $=2(2)(-2)$ $=-8$ $V(2,-8)$ Steps: 2, 6, 10



Summary:

To find the vertex by averaging the zeros, you:

a. average the zeros (x-int) to get the x-coordinate

b. sub the "x" coordinate into the equ. to get the y-coordinate.

Examples

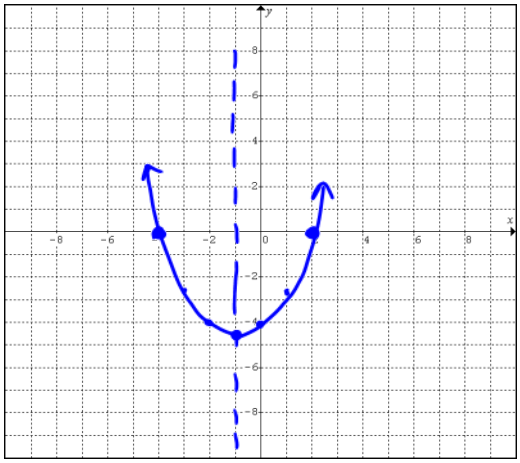
1. Given $y = \frac{1}{2}(x - 2)(x + 4)$,
   a. State the zeros: $x=2 \quad x=-4$
   b. Determine the x-coordinate of the vertex: -1 *(average the zeros)*

$$\frac{-4+2}{2} = \frac{-2}{2} = -1$$

   b. Determine the y-coordinate of the vertex: -4.5 *(substitute)*

$$y = 0.5(-1-2)(-1+4)$$
$$= 0.5(-3)(3)$$
$$= -4.5$$

   c. Graph using the zeros and vertex.



V(-1, -4.5)
Steps 0.5, 1.5, 2.5

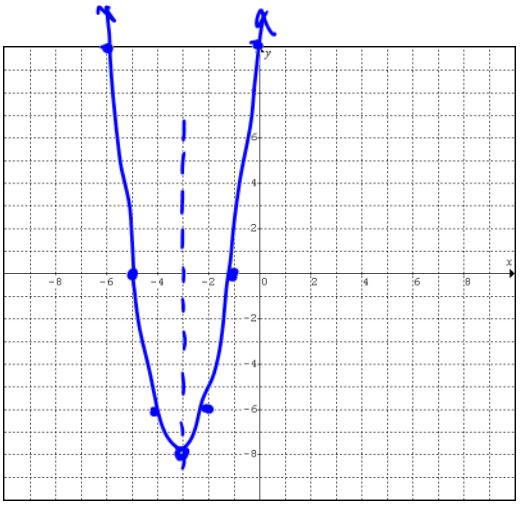
2. Given $y = 2(x + 5)(x + 1)$,
   a. State the zeros: $x=-5 \quad x=-1$
   b. Determine the x-coordinate of the vertex: -3 *(average the zeros)*

$$\frac{-5+(-1)}{2} = -3$$

   b. Determine the y-coordinate of the vertex: -8 *(substitute)*

$$y = 2(-3+5)(-3+1)$$
$$= 2(2)(-2)$$
$$= -8$$

   c. Graph using the zeros and vertex.



V(-3, -8)
Steps = 2, 6, 10

3. Determine the zeros and vertex of $y = -3x^2 + 12x$ by averaging the zeros. You can find the zeros by factoring or using the formula.

Step1: Solve the equation to find zeros

$0 = -3x^2 + 12x$ GCF $= -3x$

$0 = -3x(x-4)$

$-3x = 0$ → $x = 0$; $x - 4 = 0$ → $x = 4$; $\{0, 4\}$

Step2 "x" coordinate of vertex

$x = \frac{0+4}{2} = 2$

Step3 "y" coordinate of vertex

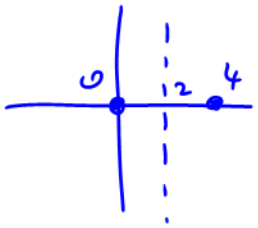
sub "2" for "x"

$y = -3(2)^2 + 12(2)$

$= -3(4) + 24$

$= -12 + 24$

$y = 12$



∴ Vertex is $(2, 12)$

Zeros are $\{0, 4\}$

4. Determine the zeros and vertex of $y = 2x^2 - 12x - 32$ by averaging the zeros. You can find the zeros by factoring or using the formula.

Step1

$y = 2(x^2 - 6x - 16)$

$y = 2(x+2)(x-8)$

zeros ⇒ $x + 2 = 0$ → $x = -2$

$x - 8 = 0$ → $x = 8$

Step2

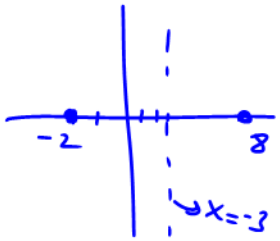
$x = \frac{-2+8}{2} = 3$

$y = 2(3)^2 - 12(3) - 32$

$= 2(9) - 36 - 32$

$= 18 - 68$

$= -50$



∴ Vertex is $(3, -50)$

Zeros are $\{-2, 8\}$

5. Determine the zeros and vertex of $y = 4x^2 - 16x + 7$ by averaging the zeros. You can find the zeros by factoring or using the formula.

Step1 Solve the equation

$0 = 4x^2 - 16x + 7$

M	A	N
28	-16	-2, -14

$0 = 4x^2 - 2x - 14x + 7$

$0 = 2x(2x-1) - 7(2x-1)$

$0 = (2x-1)(2x-7)$

$2x - 1 = 0$ → $x = 1/2$; $2x - 7 = 0$ → $x = 7/2$

Step2: $x = \frac{0.5 + 3.5}{2} = 2$

$y = 4(2)^2 - 16(2) + 7$

$= 16 - 32 + 7$

$= -9$

∴ Vertex is $(2, -9)$

Zeros are $(0.5, 3.5)$

Application: When a football is kicked with a vertical speed of 20 m/s, its height *h* metres, after *t* seconds is given by the equation $h = -5t^2 + 20t$. Determine the maximum height of the ball by averaging the zeros. You can find the zeros by factoring or using the formula.

Step1: Solve the equ.

$0 = -5t^2 + 20t$ GCF $= -5t$

$0 = -5t(t-4)$

$-5t = 0$ → $t = 0$; $t - 4 = 0$ → $t = 4$

Step2: $x = \frac{0+4}{2} = \frac{4}{2} = 2$

$y = -5(2)^2 + 20(2)$

$= -5(4) + 40$

$= -20 + 40$

$= 20$

∴ Vertex is $(2, 20)$ which means the ball reaches max height in 2 sec.