Stretched vertically by a factor of a

- If $\mathrm{a}>1$, the graph is expanded
- If $0<a<1$ the graph is compressed
- If $a<0$, the graph is reflected in the x -axis


The $a$ affects the graph $y=f(x)$ by stretching or compressing $V e c t i c o l l y \quad$ by a factor of $a$.
If the a is negative, there is a vertical reflection about the $\qquad$ "x" ax's


The d affects the graph $y=f(x)$ by translating $\square$ horizontally d units.

The c affects the graph $y=f(x)$ by translating $\qquad$ vertically c units.

The $k$ affects the graph $y=f(x)$ by stretching or compressing horizentally_b by actor of $\frac{1}{k}$.
If the $k$ is negative, there is a horizontal reflection about the

*Does the order of transformations matter?
(1) Reflection
(2) Stretch/Compression
(3) Translation

$$
\begin{aligned}
& \text { YES, we follow } f[-3(x-2)]+1 \\
& \text { ex: } y=2 f[
\end{aligned}
$$

$R$ : horizontal reflection about " $y$ " axis $s=$ stretched vertically by a factor of 2 compressed haizontally by a factor of $1 / 3$ $T=$ translated 2 units left and lunit up.

Graphing: Mapping Notation
If you can successfully figure out the mapping notation for the transformed function, the graphing part is rather easy. Here is the formula below.

$$
y=a f[k(x-d)]+c
$$

$$
(x, y) \rightarrow\left(\frac{x}{k}+d, a y+c\right)
$$



$$
\begin{aligned}
& (x, y) \rightarrow\left(-\frac{x}{2}+1,3 y+1\right) \\
& (4,4) \rightarrow\left(-\frac{4}{2}+1,3(4)+1\right)=(-1,13)
\end{aligned}
$$

Ex State the mapping notation for: $y=-\frac{1}{2} f\left[-\frac{1}{2}(x+1)\right]-1(x, y) \longrightarrow\left(-2 x-1, \frac{-y}{2}-1\right)$

$$
(1,2) \longrightarrow\left(-2 \cdot 1-1,-\frac{2}{2}-1\right)=(-3,-2)
$$

Example 1 - Sketching Graphs of Transformed Functions

1. Given the graph of $f(x)=|x|$, state the mapping notation for $g(x)=-2 f(x-6)+4$ using transformations.

$$
\begin{gathered}
(x, y) \rightarrow(x+6,-2 y+4) \\
A(-3,3) \rightarrow(-3+6,-2(3)+4)=A^{\prime}(3,-2) \\
B(0,0) \rightarrow(0+6,-2(0)+4)=B^{\prime}(6,4) \\
C(3,3) \rightarrow(3+6,-2(3)+4)=C(9,-2) \\
D=\{x \in \mathbb{R}\} \quad R=\{y \in \mathbb{R} \mid y \leqslant 4\}
\end{gathered}
$$



$$
\begin{aligned}
& \text { 2. Using transformations, sketch th -6 } \\
& \text { Hint: Rewrite } 2 x+4 \text { in factor } \\
& \text { horizontal translation. } \\
& f(x)=\sqrt{2(x-2)}+1 \\
& (x, y) \rightarrow\left(\frac{x}{2}+2, y+1\right) \\
& A(0,0) \rightarrow\left(\frac{0}{2}+2,0+1\right)=A^{\prime}(2,1) \\
& B(1,1) \rightarrow\left(\frac{1}{2}+2,1+1\right)=B^{\prime}(2.5,2) \\
& C(4,2) \rightarrow\left(\frac{4}{2}+2,2+1\right)=C^{\prime}(4,3) \\
& D(9,3) \rightarrow\left(\frac{9}{2}+2,3+1\right)=D^{\prime}(6.5,4)
\end{aligned}
$$

Example 2 - Writing Equations of Transformed Functions

1. The function $y=f(x)$ has been transformed into $y=a f(k(x-d))+c$. Write the following in the appropriate form:
(a) a vertical compression by a factor of $1 / 2$, a reflection in the $x$-axis and a translation 3 units right.

$$
\begin{aligned}
& a=1 / 2 \\
& y=-\frac{1}{2} f[x-(+3)] \Rightarrow y=-\frac{-1}{2} f(x-3)
\end{aligned}
$$

(b) a vertical stretch by a factor of 3 , a horizontal stretch by a factor of 2 , a translation left 5 and up 4 , and a reflection in the $y$-axis. $\rightarrow$ (horizontal refl)

$$
\begin{aligned}
& a=3 \\
& k=-\frac{1}{2} \\
& d=-5
\end{aligned} \quad y=3 f\left[-\frac{1}{2}(x-(-5)]+4 \Rightarrow y=3 f\left[-\frac{1}{2}(x+5)\right]+4\right.
$$

Practice Transformations Given an Equation
Graph each of the following functions by:
a) Graphing the base function first. $\left(y=x^{2}, y=\sqrt{x}, y=x^{3}, y=|x|, y=\frac{1}{x}\right)$
b) Listing the transformations. asymptotes
c) Applying the transfo

1) $y=2(x+1)^{2}-1$

Base $y=x^{2}$
$R=$ none
$S=$ stretched vertically bafo 2

$$
\begin{aligned}
& T=1 \text { unit left } \\
& 1 \text { unit down } \\
&(x, y) \longrightarrow(x-1,2 y-1)
\end{aligned}
$$

$A(-2,4) \rightarrow(-2-1,2 \cdot 4-1)=A^{\prime}(-3,7)$
$B(-1,1) \longrightarrow(-1-1,2 \cdot 1-1)=B^{\prime}(-2,1)$

$$
C(0,0) \rightarrow(0-1,2 \cdot 0-1)=c^{\prime}(-1,-1)
$$



Bax: $y=\frac{1}{x}$

$$
R=\text { none }
$$

$S=$ stretched vert bafo 2
$T=2$ units left, up

$$
(x, y) \rightarrow(x-2,2 y+1)
$$ Bax $y=x^{3}$

$$
A(1,1) \rightarrow A^{\prime}(-1,3)
$$

$$
B(-1,-1) \rightarrow B^{\prime}(-3,-1)
$$



$$
\begin{aligned}
& R=\text { none } \\
& S=\text { horizontal comp. bofo } 1 / 2 \\
& T=1 \text { unit right, } 1 \text { up } \\
& (x, y) \rightarrow\left(\frac{x}{2}+1, y^{\prime}-1\right) \\
& A(1,1) \rightarrow\left(\frac{1}{2}+1,1-1\right)=(1.5,0) \\
& B(0,0) \rightarrow\left(\frac{0}{2}+1,0-1\right)=(1,-1) \\
& C(-1,-1) \rightarrow\left(\frac{-1}{2}+1,-1-1\right)=C^{\prime}(0.5,-2) \\
& D(-2,-8) \rightarrow\left(\frac{-2}{2}+1,-8-1\right)=D^{\prime}(0,-9)
\end{aligned}
$$



4）

$$
\begin{aligned}
& y=-\sqrt{x-2}+1 \\
& y=\sqrt{x}
\end{aligned}
$$

$R=$ vertical reflection about＂x＂axis $S=$ none $T=2$ right ， 1 up

$$
\begin{aligned}
& =2 \text { right, } \\
& (x, y) \rightarrow(x+2,-y+1) \\
& A(0,0) \xrightarrow{(x)} \rightarrow(1+2,-0+1)=(2,1)=B^{\prime}(3,0) \\
& B(1,1) \rightarrow(4+2,-2+1)=C^{\prime}(6,-1) \\
& C(4,2) \longrightarrow y=\sqrt{x}
\end{aligned}
$$

7）$y=\left|\frac{1}{2} x-\frac{1}{2}\right|$

$$
y=\left|\frac{1}{2}(x-1)\right|
$$

$R=$ none
$S=$ horizontal str．bafo 2
$T=\alpha$ right

$$
\begin{aligned}
& (x, y) \rightarrow(2 x+1, y) \\
& A(-1,1) \rightarrow(2 \cdot-1+1,1)=A^{\prime}(-1,1) \\
& B(0,0) \rightarrow(2 \cdot 0+1,0)=B^{\prime}(1,0) \\
& C(1,1) \rightarrow(2 \cdot 1+1,1)=C^{\prime}(3,1)
\end{aligned}
$$



5）$y=2 \sqrt{3 x}$
$y=\sqrt{x}$
$R=$ nome
$S=$ stretched vertically bafo 2
horizontally compressed boffo 1／3
T：nome
$(x, y) \rightarrow\left(\frac{x}{3}, 2 y\right)$
$A(0,0) \rightarrow\left(\frac{0}{3}, 2.0\right)=A^{\prime}(0,0)$
$B(1,1) \rightarrow\left(\frac{1}{3}, 2.1\right)=B^{\prime}\left(\frac{1}{3}, 2\right)$

$$
C(4,2) \rightarrow\left(\frac{4}{3}, 2.2\right)=C\left(\frac{4}{3}, 4\right)
$$



$R=$ none
$S=$ vertical stretch bate 3
$T=1$ 恰t，lug

$$
(x, y) \xrightarrow{ }(x-1,3 y+1)
$$

$$
A(-1,1) \rightarrow(-1-1,3 \cdot 1+1)=A^{\prime}(-2,4)
$$

$$
B(0,0) \rightarrow(0-1,30+1)=B^{\prime}(-1,1)
$$

$$
C(1,1) \rightarrow(1-1,3 \cdot 1+1)=C^{\prime}(0,4)
$$

9）$y=2 \sqrt{2 x+2}-2 \quad y=2 \sqrt{2(x+1)}-2$ $R=$ no re
$y=x^{3}$
$R=$ vertical refile．about＂$x$＂ horizontal＂＂＂$y$＂
$S=$ horizontal stretch beta 2 $T=\alpha$ left

$$
(x, y) \rightarrow(-2 x-1,-y)
$$

$A(1,1) \rightarrow(-2 \cdot 1-1,-1)=A^{\prime}(-3,-1)$
$B(0,0) \rightarrow(-2 \cdot 0-1,-0)=B^{\prime}(-1,0)$
$C(-1,-1) \rightarrow(-2(-1)-1) ;(-1)=C^{\prime}(1,1)$
$S=$ vertical stretch buff 2 horizontal comp bafo $1 / 2$
$T$ ：$\&$ left， 2 down

$$
\begin{gathered}
\\
\\
(-3,-1) \\
(-1,0) \\
1,1) \\
A(0,0) \rightarrow\left(\frac{0}{2}-1,2(0)-2\right)=A^{\prime}(-1,-2) \\
B(1,1) \rightarrow\left(\frac{1}{2}-1,2 \cdot 1-2\right)=B^{\prime}(-0.5,0) \\
C(4,2) \rightarrow\left(\frac{4}{2}-1,2 \cdot 2-2\right)=C^{\prime}(1,2)
\end{gathered}
$$



## 11 Academic

Day 8: Combinations of Transformations

Date:
Unit 1: Intro to Functions

## Practice Transformations Given a Graph

List the transformations.


$$
\begin{array}{lll}
\text { 5) } y=f(3 x-6) \\
(x, y) & \rightarrow\left(\frac{x}{3}+2, y\right) & \text { 6) } y=f(-2 x+4) \quad y=f[3(x-2)] \\
& & (x, y) \rightarrow\left(\frac{-x}{2}+2, y\right) \\
A(-6,6) \rightarrow A^{\prime}(0,6) & D(2,-2) \rightarrow D^{\prime}\left(\frac{8}{3},-2\right) & A(-6,7) \rightarrow A^{\prime}(5,7) \\
B(-2,2) \rightarrow B^{\prime}\left(\frac{4}{3},-2\right) & E(4,7) \rightarrow E^{\prime}\left(\frac{10}{3}, 7\right) & B(-3,-4) \rightarrow B^{\prime}(3.5,-4) \\
C(0,0) \rightarrow C^{\prime}(2,0) & & C(-1,2) \rightarrow C^{\prime}(2.5,2)
\end{array}
$$

R: none


7) $y=-2 f(x-3)+1(x, y) \rightarrow(x+3,-2 y+1)$
$R=$ vertical reflection $A(-3,2) \rightarrow A^{\prime}(0,-3)$
8) $y=\frac{1}{2} f\left(\frac{1}{3} x-2\right)$

$$
y=\frac{1}{2} f\left[\frac{1}{3}(x-6)\right]
$$ about " $x$ " axis $B(0,5) \rightarrow B^{\prime}(3,-9)$

$$
\begin{aligned}
(x, y) & \rightarrow\left(3 x+6, \frac{y}{2}\right) \\
A(-6,-4) & \rightarrow A^{\prime}(-12,-2)
\end{aligned} \quad D(3,2) \rightarrow D^{\prime}(15,1)
$$

$$
\begin{aligned}
S= & \text { vertical stretch } C(3,2) \rightarrow C^{\prime}(6,-3) \\
& \text { bolo } 2
\end{aligned}
$$


$C(0,0) \rightarrow C^{\prime}(6,0)$


