Stretched vertically by a factor of a
- If a > 1, the graph is expanded
- If 0 < a < 1 the graph is compressed
- If a < 0, the graph is reflected in the x-axis

Translated vertically c units
- If c > 0, the graph shifts up
- If c < 0, the graph shifts down

$$y = af\left(k\left(x-d\right)\right)+c$$

Stretched horizontally by a factor of 1/k
- If k >1, the graph is compressed
- If 0 < k < 1 the graph is expanded
- If k < 0, the graph is graph is reflected in the y-axis

Translated horizontally d units
- If the sign is negative, the graph shifts to the right
- If the sign is positive, the graph shifts to the left

The a affects the graph $y = f\left(x\right)$ by stretching or compressing ____________________ by a factor of a.

If the a is negative, there is a vertical reflection about the ___________________.

The d affects the graph $y = f\left(x\right)$ by translating _______________________ d units.

The c affects the graph $y = f\left(x\right)$ by translating _______________________ c units.

The k affects the graph $y = f\left(x\right)$ by stretching or compressing _____________________ by a factor of $\frac{1}{k}$.

If the k is negative, there is a horizontal reflection about the __________________.

***Does the order of transformations matter?**

Graphing: Mapping Notation

If you can successfully figure out the mapping notation for the transformed function, the graphing part is rather easy. Here is the formula below.

$$\boldsymbol{y = af[k(x-d)] + c}$$

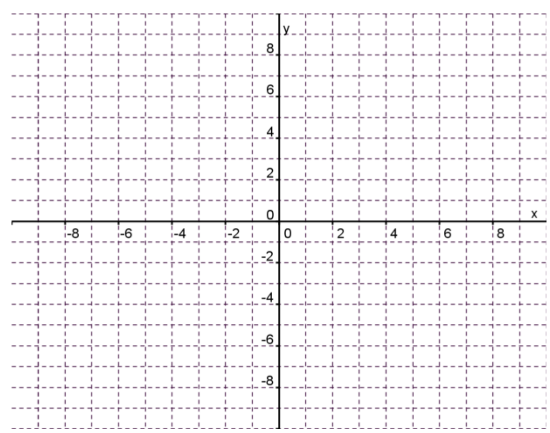
$$\textbf{(x,y)} \rightarrow \left(\frac{x}{k} + d, ay + c\right)$$

Ex1 State the mapping notation for: $y = 3f[-2(x-1)] + 1$

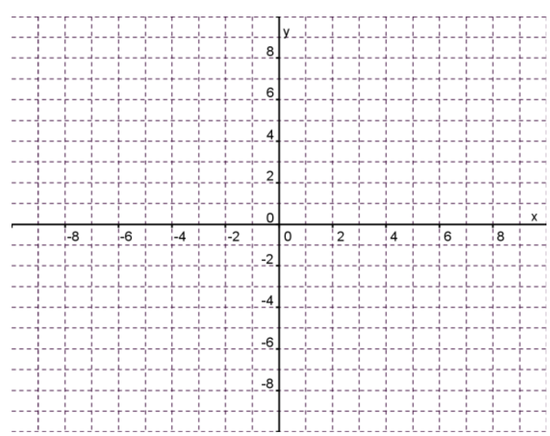
Ex2 State the mapping notation for: $y = -\frac{1}{2}f\left[-\frac{1}{2}(x+1)\right] - 1$

Example 1 – Sketching Graphs of Transformed Functions

1. Given the graph of $f(x) = |x|$, state the mapping notation for $g(x) = -2f(x-6) + 4$ using transformations.



2. Using transformations, sketch the graph of $f(x) = \sqrt{2x+4}$.
   Hint: Rewrite $2x+4$ in factored form to determine the horizontal translation.



Example 2 – Writing Equations of Transformed Functions

1. The function $y = f(x)$ has been transformed into $y = af(k(x - d)) + c$. Write the following in the appropriate form:
   (a) a vertical compression by a factor of ½ , a reflection in the x-axis and a translation 3 units right.

   (b) a vertical stretch by a factor of 3, a horizontal stretch by a factor of 2, a translation left 5 and up 4, and a reflection in the y-axis.

Practice Transformations Given an Equation

Graph each of the following functions by:

a) Graphing the base function first. $\left(y = x^2, y = \sqrt{x}, y = x^3, y = |x|, y = \frac{1}{x} \right)$

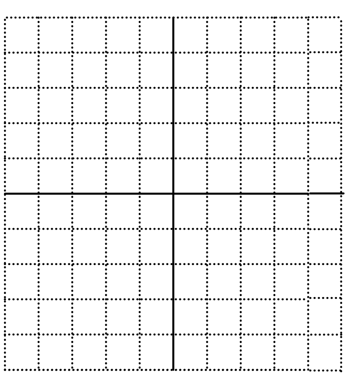
b) Listing the transformations.

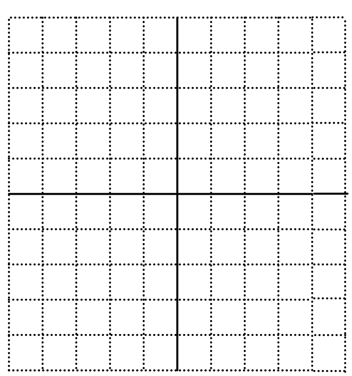
c) Applying the transformations to the base function.

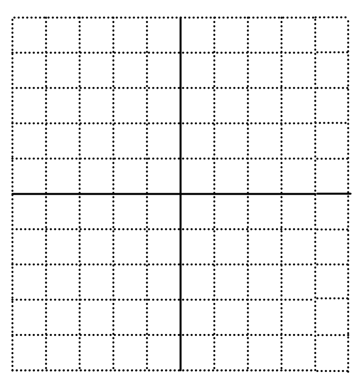
1) $y = 2(x+1)^2 - 1$

2) $y = \frac{2}{x+2} + 1$

3) $y = (2x-2)^3 - 1$



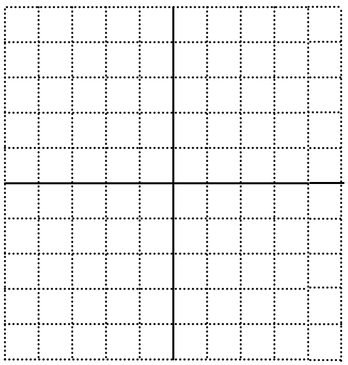


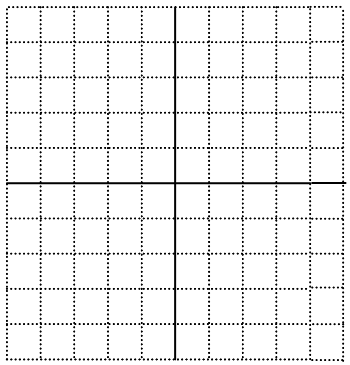


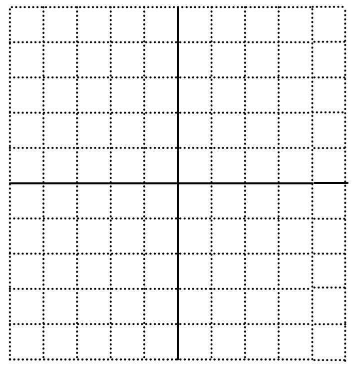
4) $y=-\sqrt{x-2}+1$

5) $y=2\sqrt{3x}$

6) $y=3|x+1|+1$



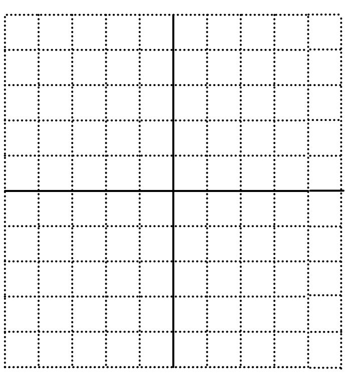


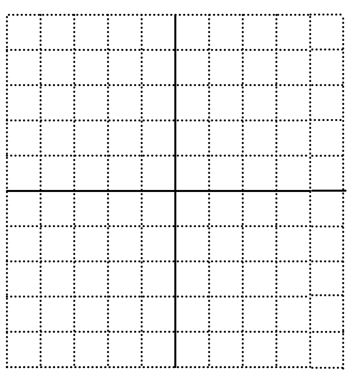


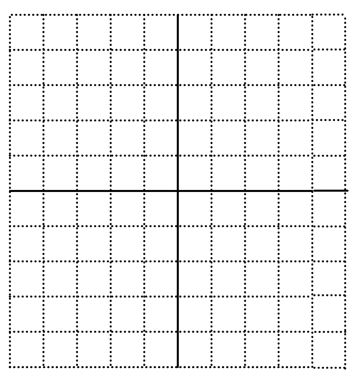
7) $y=\left|\frac{1}{2}x-\frac{1}{2}\right|$

8) $y=-\left(-\frac{1}{2}(x+1)\right)^3$

9) $y=2\sqrt{2x+2}-2$







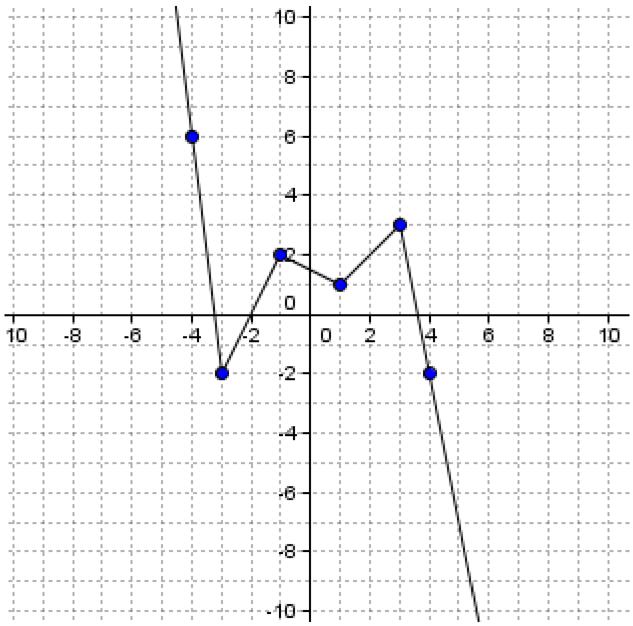
Practice Transformations Given a Graph

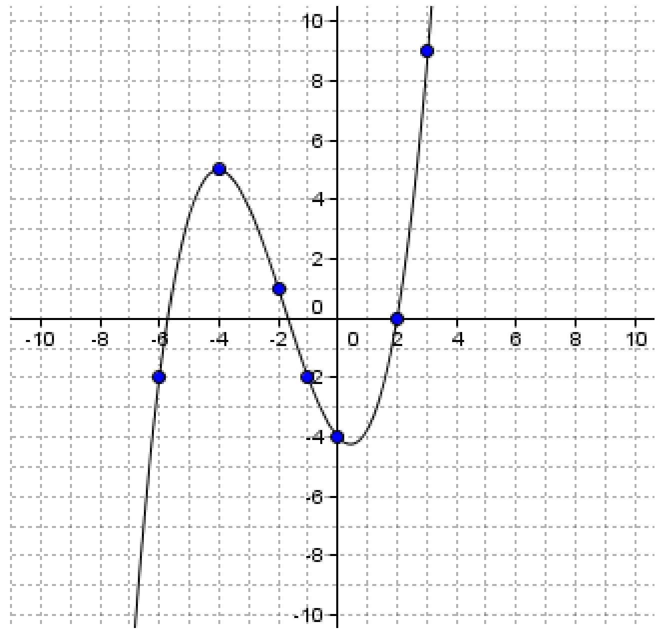
List the transformations.

Apply the transformations to key points on the graph.

1) $y = 3g\left(-2(x-1)\right)+1$

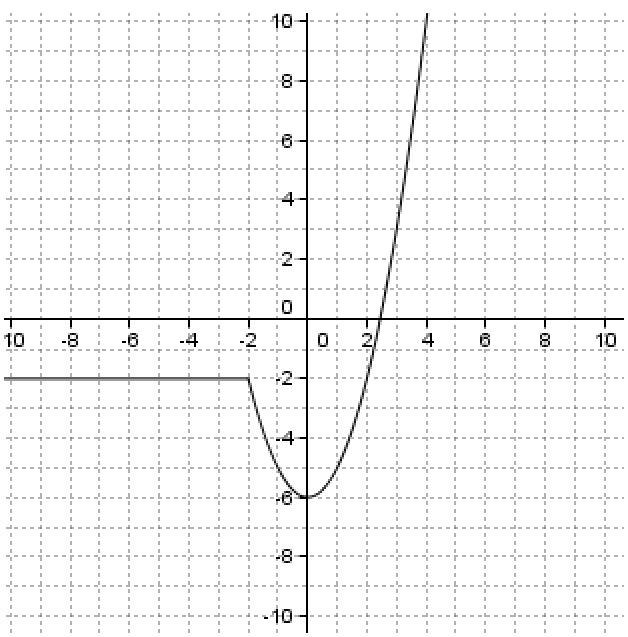
2) $y = -f\left(2(x+1)\right)$

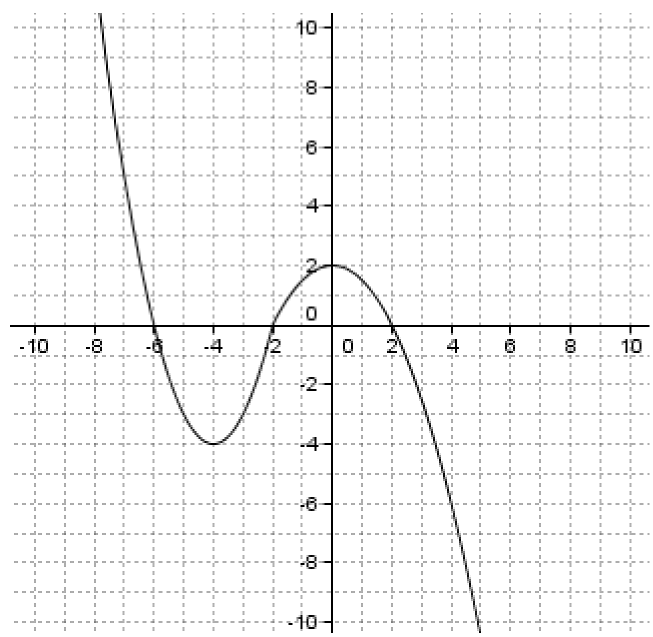




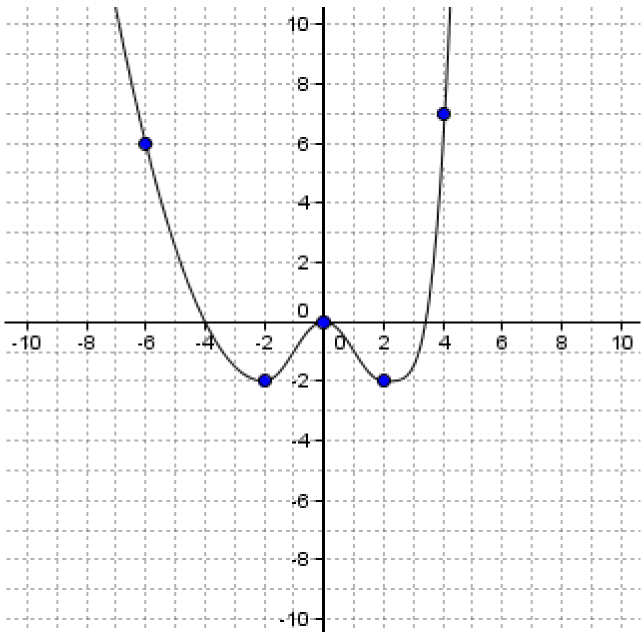
3) $y = \frac{1}{2}h(x+1)-2$

4) $y = -\frac{1}{2}f(2x+4)-1$

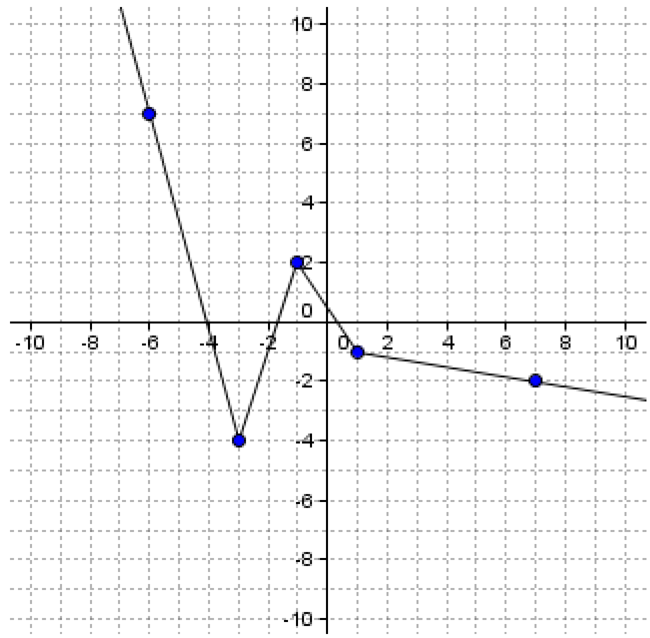




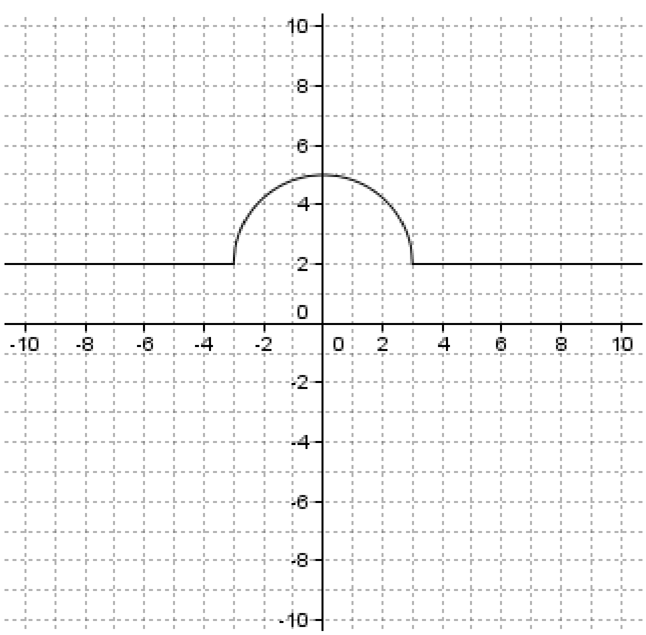
5) $y = f(3x-6)$



6) $y = f(-2x+4)$



7) $y = -2f(x-3)+1$



8) $y = \frac{1}{2} f\left(\frac{1}{3}x-2\right)$