MAP4C **Date: ____________**

Day 8: Applications of Exponential Equations **Unit 6: Algebraic Models**

Exponential relations are modelled by the exponential equation $y = ab^x$,
where
a is the initial amount
b is the growth\decay factor
$b > 1$ models growth
$0 < b < 1$ models decay

Real-world applications of exponential growth or decay may require solving the equation $y = ab^x$ for x

KEY WORDS
$y = ab^x$
initial
growth/decay
growth
decay

***EXAMPLE 1**: Guess & Check*

A city's population can be modelled by the equation $P = 1.4(1.0415)^t$, where P represents the population in millions t years after 1985. In which year did the population first exceed 3 million?

Beginning population: 1.4 million Ending population: 3 million

Population is growing at a rate of: 0.0415 or 4.15% *Rate* = Growth Factor − 1

$1.0415 - 1$

0.0415

Sub in the info given: $\frac{3}{1.4} = \frac{1.4(1.0415)^t}{1.4}$ ① ÷1.4 BS

Simplify:

$2.1429 = (1.0415)^t$

LS RS

Use a ***TABLE OF VALUES*** to ***GUESS & CHECK*** a solution *Simply try random t values.*

t is between 15 and 20

t	RS
10	1.5
15	1.84
20	2.26

→

t	RS
10	1.5
15	1.84
16	1.92
17	1.99
18	2.08
19	2.16

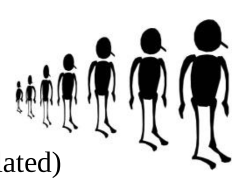
∴ t is between 18 and 19 years

<u>TWO SPECIAL EXPONENTIAL GROWTH & DECAY EQUATIONS</u>

DOUBLING: $A = A_0(2)^{t/d}$

A_0 = Initial Amount

A = Final Amount



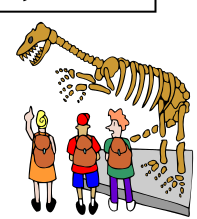
t = time (measured or calculated)

$\boldsymbol{d}$ = time it takes to double

HALF-LIFE: $A = A_0(0.5)^{t/h}$

A_0 = Initial Amount

A = Final Amount



t = time (measured or calculated)

$\boldsymbol{h}$ = time needed to reduce initial amount by half.

EXAMPLE 3: Solving an Application Involving Doubling Time

A bacteria culture doubles in size approximately every 14 hours. Suppose this bacteria culture started with 100 individual bacteria. How long will it take for the bacteria population to reach 1000 individuals? Give your answer to one decimal place.

$A_o = 100$
$A = 1000$
$t = ?$
$d = 14$

Step 1

$A = A_o(2)^{t/d}$

$1000 = 100(2)^{t/14}$ divide BS by 100

$\div 100 \quad \div 100$

$10 = 2^{t/14}$

$10 = 2^{n}$

Step 2

n	2^n
3	$2^3 = 8$
3.1	$2^{3.1} = 8.6$
3.2	$2^{3.2} = 9.2$
3.3	$2^{3.3} = 9.8$
3.4	$2^{3.4} = 10.6$

Step 3

$\frac{t}{14} = n$

$14 \times \frac{t}{14} = 3.3 \times 14$

$t = 46.2$

∴ It'll take 46.2 hours.

EXAMPLE 4: Solving Applications Involving Half-Life

Caffeine has a half-life of approximately 5 hours. Suppose you drink a cup of coffee that contains 200 mg of caffeine. How long will it take until there is less than 10 mg of caffeine left in your bloodstream?

$A = 10$
$A_o = 200$
$t = ?$
$h = 5$ hours

Step 1

$A = A_o(0.5)^{t/h}$

$\frac{10}{200} = \frac{200(0.5)^{t/5}}{200}$ $\div 200$

$0.05 = (0.5)^{t/5} \rightarrow n$

n	0.5^n
2	$0.5^2 = 0.25$
3	$0.5^3 = 0.125$
4	$0.5^4 = 0.0625$
5	$0.5^5 = 0.03125$

$0.5^{4.3} = 0.05$

Step 3

$5 \times \frac{t}{5} = 4.3 \times 5$

$t = 21.5$

∴ It'll take 21.5 hours.