# Lesson: Regular Payments of an Annuity

**_RECALL: FUTURE VALUE_**

*Use to find the value* ***at the end of an annuity*** *(after all deposits are made & interest is accrued)*

$$A = \frac{R\left[(1+i)^n - 1\right]}{i}$$

**_RECALL: PRESENT VALUE_**

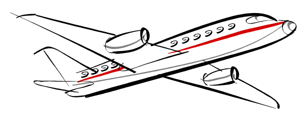
*Use to find the money needed* ***at the beginning of an annuity*** *to provide regular annuity payments*

$$PV = \frac{R\left[1-(1+i)^{-n}\right]}{i}$$

## *Calculating the Regular Payment of an Annuity*

When we know the future value or the present value of annuity, we can ***rearrange the formula*** to ***ISOLATE R*** to ***solve for the regular payment***. Remember, rearranging formulas means you do BEDMAS backwards.

### *EXAMPLE 1 Determining Payments given the Amount (Future Value)*



Bella wants to save $6000 for a trip she plans to take in 5 years. What **regular deposit** should she make at the end of every 6 months into an account that earns 6% per year compounded semi-annually?

Type : compounded semi-annually

A : $6000

R : ?

i : 6%/1 year = 0.06 ÷ 2 = 0.03

n: 5 years x 2 = 10 periods

$$A = \frac{R\left((1+i)^n - 1\right)}{i}$$

$$6000 = \frac{R\left((1+0.03)^{10}-1\right)}{0.03}$$ using all the brackets, evaluate RS

$$\frac{6000}{11.4638} = \frac{R \cdot 11.4638}{11.4638}$$ divide BS by 11.4638

$$\boxed{\$523.38 = R}$$

∴ Bella needs to make a regular payments of $523.38 every 6 months.

### *EXAMPLE 2 Determining Payments Given the Present Value*

Niloufar borrows $1200 (present) from an electronics store to buy a computer. She will repay the loan in equal monthly payments over 3 years, starting 1 month from now. She is charged interest at 12.5% per year compounded monthly. How much is Niloufar's monthly payment?

Type : compounded monthly

Pv: $1200

R : ?

I : 12.5%/1year = 0.125÷12 → kept it like that b/c didn't want to round

n : 3 years x 12 = 36

$$PV = \frac{R\left[1-(1+i)^{-n}\right]}{i}$$

$$1200 = \frac{R\left(1-(1+0.125\div 12)^{-36}\right)}{(0.125\div 12)}$$ Evaluate RS Input all the brackets

$$\frac{1200}{29.8921} = \frac{R \cdot \cancel{29.8921}}{\cancel{29.8921}}$$

$$40.14 \doteq R$$

∴ Niloufar needs to pay $40.14 every month for 3 years.

∴ EXTRA: She will pay 40.14 x 36 = $1445.04 in total; thus the interest she will pay will be $245.04.

***EXAMPLE 3 Comparing Loan Options*** → Present Value

Angela borrows $9500 to buy a car. She can repay her loan in 2 ways.

- **Option A**: 36 monthly payments at 6.9% per year compounded monthly
- **Option B**: 60 monthly payments at 8.9% per year compounded monthly

**a)** What is Angela's monthly payment for each option?

Option A

Type: compound monthly

PV: $9500

R: ?

i: 6.9%/1 year = 0.069 ÷ 12

n: 36

$$PV = \frac{R\left(1-(1+i)^{-n}\right)}{i}$$

$$9500 = \frac{R\left(1-(1+0.069\div 12)^{-36}\right)}{(0.069\div 12)}$$

$$\frac{9500}{32.4345} = \frac{R \cdot 32.4345}{32.4345} \qquad R \cong 292.90$$

Option B

Type: compound monthly

PV: $9500

R: ?

i: 8.9%/1 year = 0.089 ÷ 12

n: 60

$$9500 = \frac{R\left(1-(1+0.089\div 12)^{-60}\right)}{(0.089\div 12)}$$

$$\frac{9500}{48.2862} = \frac{R \cdot 48.2862}{48.2862}$$

$$R \cong 196.74$$

**b)** How much interest does Angela pay for each option?

Interest = Total Payments − $ Borrowed

OPTION A

Total Payment = 292.90 × 36
= $10544.40

I = 10544.40 − 9500
= $1044.40

OPTION B

Total Payment = 196.74 × 60
= $11804.40

I = 11804.40 − 9500
= $2304.40

**c)** Give a reason why Angela might choose each option.

Option A is better financially for Angela if she can afford almost $300 monthly payments. She will pay less interest.

Option B is better for Angela if she cannot afford almost $300 monthly payments. She will also have to pay insurance at least $100 a month, gas $200 plus other maintenance costs.