Solving Exponential Equations

An ***exponential equation*** is one in which the unknown is contained within an exponent or exponents.

As with all types of equations, algebra can be used to determine an exact solution for an exponential equation. When the powers on either side of the equation have the same base, the exponents can be set equal and the resulting equation solved.

In other words:

if $a^x = a^y$
then $x = y$

Ex1. Solve each of the following:

a) $3^{2x} = 81$

$3^{2x} = 3^4$

$2x = 4$

$\boxed{x=2}$ ✓

81 | 3
27 | 3
9 | 3
3 | 3
1 |

b) $5^{2x-1} = \frac{1}{125}$

$5^{2x-1} = (125)^{-1}$

$5^{2x-1} = (5^3)^{-1}$

if $5^{2x-1} = 5^{-3}$

then $2x-1 = -3$

$2x = -2$

$\boxed{x=-1}$

125 | 5
25 | 5
5 | 5
1 |

5^3

c) $3^x = 9^{x-1}$

$3^x = (3^2)^{x-1}$

$3^x = 3^{2(x-1)}$

$3^x = 3^{2x-2}$

$x = 2x-2$

$\boxed{2=x}$

d) $\frac{4(2^x)}{4} = \frac{32}{4}$ 8 → simplify first

$2^x = 8$

$2^x = 2^3$

$\boxed{x=3}$

e) $2^{x+2} - 2^x = 48$; let's express it in factored form

$2^x \cdot 2^2 - 2^x = 48$;

$\underbrace{4 \cdot 2^x - 2^x} = 48$: 2^x is a variable
You have $4 \cdot 2^x$ and are subtracting $1 \cdot 2^x$
Thus we have $3 \cdot 2^x$

$\frac{3 \cdot 2^x}{3} = \frac{48}{3}$

$2^x = 16$ Express 16 as power

$2^x = 2^4$

$\boxed{x=4}$

$\therefore$ x is 4

f) $2^{2x} - 33(2^x) + 32 = 0$

$(2^x)^2 - 33(2^x) + 32 = 0$ Let "a" be 2^x

$a^2 - 33a + 32 = 0$

$(a-1)(a-32) = 0$

M	A	N
32	33	-1, -32

$a = 1$ $a = 32$

$2^x = 1$ $2^x = 32$

$\boxed{x=0}$ $2^x = 2^5$

$\boxed{x=5}$

32 | 2
16 | 2
8 | 2
4 | 2
2 | 2
1 |

2^5

$\therefore$ x is either 0 or 5.

Ex2. Solve each of the following:

a) $2^{x+1} + 3(2^x) = 80$

$2^x \cdot 2^1 + 3(2^x) = 80$

$2(2^x) + 3(2^x) = 80$

To see it better replace 2^x with "a"

$2(a) + 3(a) = 80$

$\frac{5(a)}{5} = \frac{80}{5}$

$a = 16$

Now replace a with 2^x

$2^x = 2^4$

$x = 4$

$\therefore x$ is 4

b) $3^{x+5} + 3^{x+4} = 36$

$3^x \cdot 3^5 + 3^x \cdot 3^4 = 36$

Let "a" rep 3^x

$243(3^x) + 81(3^x) = 36$

$243(a) + 81(a) = 36$

$\frac{324(a)}{324} = \frac{36}{324}$ → keep the answer as fraction

$a = 1/9$

$a = 9^{-1}$

$3^x = 3^{-2}$

$\boxed{x = -2}$

$\therefore x$ is -2

c) $7(2^{3x}) - 3 = 445$

$\frac{7(2^{3x})}{7} = \frac{448}{7}$

$2^{3x} = 64$

$2^{3x} = 2^6$

$3x = 6 \rightarrow \boxed{x = 2}$

64	2
32	2
16	2
8	2
4	2
2	2
1	

2^6

d) $5^{x^2-6x} = 625^{2-x}$

$5^{x^2-6x} = (5^4)^{2-x}$

$5^{x^2-6x} = 5^{8-4x}$

$x^2 - 6x = 8 - 4x$

$x^2 - 2x - 8 = 0$

$(x+2)(x-4) = 0$

$\therefore x = -2$ or $x = 4$

e) $5(3^{x^2-x}) + 2 = 3647$

$5(3^{x^2-x}) = 3645$

$3^{x^2-x} = 729$

$3^{x^2-x} = 3^6$

$x^2 - x = 6$

$x^2 - x - 6 = 0$

$(x+2)(x-3) = 0$

$x = 3$ or $x = -2$

729	3
243	3
81	3
27	3
9	3
3	3
1	

3^6

f) $216^{x-4} = \sqrt{1296}$

36	6
6	6
1	

216	6
36	6
6	6
1	

$6 \times 6 \times 6 = 6^3$

$(6^3)^{x-4} = 36$

if $\boxed{6}^{3x-12} = \boxed{6}^2$

then $3x - 12 = 2$

$3x = 14$

$\boxed{x = 14/3}$

$\therefore x$ is $14/3$

g) $\frac{8^{2x}}{4^{x-1}} = 2^{x+1}$

① $\frac{(2^3)^{2x}}{(2^2)^{x-1}} = 2^{x+1}$ ⇒ ② $\frac{2^{6x}}{2^{2x-2}} = 2^{x+1}$ ⇒ ③ $2^{6x-2x+2} = 2^{x+1}$

④ $2^{4x+2} = 2^{x+1}$