9.4 Maximize the Volume of a Square-Based Prism

Have you ever been restricted by the amount of material you had to finish a job? This may have happened when you were wrapping a gift or packaging food. In the packaging industry, it may be important to make a carton with the greatest possible volume from the cardboard that is available. This involves determining the maximum, or optimal, volume for a given surface area.

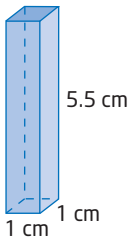
Investigate

How can you compare the volumes of square-based prisms with the same surface area?

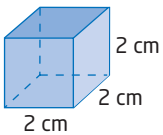
Method 1: Pencil and Paper

1. Each of the square-based prisms has a surface area of 24 cm^2. Calculate the area of the base and the volume of each prism. Record your data in a table.

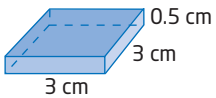
Prism 1



Prism 2



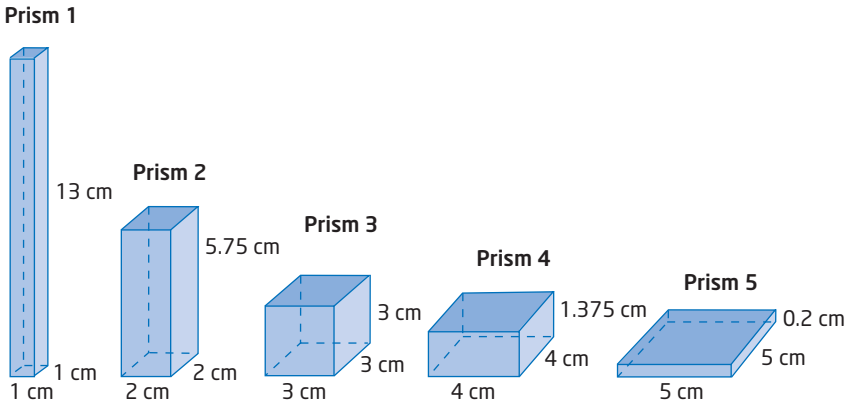
Prism 3



Prism Number	Side Length of Base (cm)	Area of Base (cm^2)	Surface Area (cm^2)	Height (cm)	Volume (cm^3)
1			24		
2			24		
3			24		

2. Which square-based prism has the maximum volume? Describe the shape of this prism compared to the others.

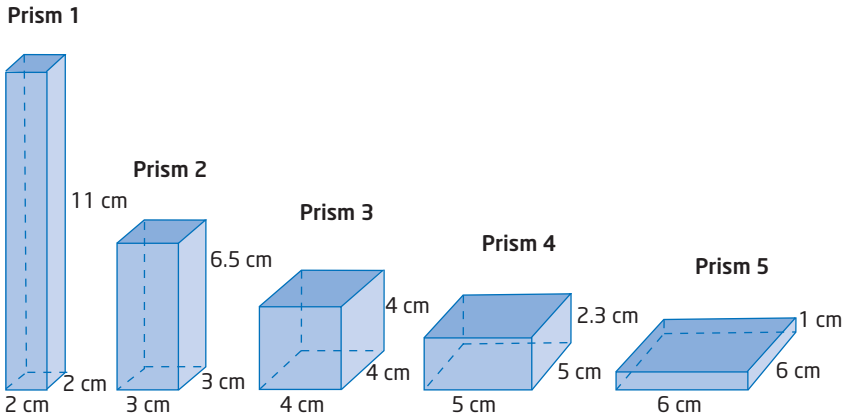
3. Each of the square-based prisms has a surface area of 54 cm^2.



a) Predict the dimensions of the square-based prism with maximum volume if the surface area is 54 cm^2.

b) Test your prediction by completing a similar table to the one in step 1.

4. Repeat step 3 for a square-based prism with surface area 96 cm^2.



5. **Reflect** What conclusion can you make about the maximum volume of a square-based prism with a given surface area?

Method 2: Use a Spreadsheet

Tools
- spreadsheet software
- computers

Use a spreadsheet to examine the volume of different square-based prisms with a fixed surface area of 24 cm^2.

1. Create a spreadsheet with formulas as follows.

	A	B	C	D	E
1	Side Length of Base (cm)	Area of Base (cm^2)	Surface Area (cm^2)	Height (cm)	Volume (cm^3)
2	1	=A2^2	24	=(C2-2*B2)/(4*A2)	=B2*D2
3	=A2+1	=A3^2	24	=(C3-2*B3)/(4*A3)	=B3*D3
4					

2. You can find the surface area of a square-based prism by calculating 2(area of base) + 4(area of sides). The surface area of the prism is always 24 cm^2. So, the height can be found using the expression (24 − 2*(area of base))/(4*(side length of base)). Explain why.

3. Use **Fill Down** to complete the spreadsheet. What dimensions result in the greatest volume? Describe the shape of this square-based prism.

4. **a)** Predict the dimensions of the square-based prism with maximum volume if the surface area is 54 cm^2.

   b) Check your prediction by changing the surface area value in the spreadsheet.

5. Repeat step 4 for a square-based prism with surface area 96 cm^2.

6. **Reflect** What conclusion can you make about the maximum volume of a square-based prism with a given surface area?

7. Save your spreadsheet for future use.

Example Maximize the Volume of a Square-Based Prism

a) Determine the dimensions of the square-based prism with maximum volume that can be formed using 5400 cm^2 of cardboard.

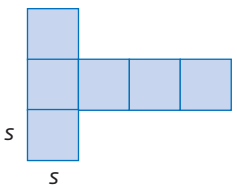
b) What is the volume of the prism?

Solution

a) Given the surface area of a square-based prism, the prism with the maximum volume is in the shape of a cube. This means that the sum of each of the six square faces of the cube must be 5400 cm^2.

Let s represent the length of each side of the cube.

$$\begin{aligned} SA &= 6s^2 \\ 5400 &= 6s^2 \\ 900 &= s^2 \\ \sqrt{900} &= s \\ 30 &= s \end{aligned}$$



The square-based prism with maximum volume is a cube with side length 30 cm.

b) Use the formula for the volume of a cube.

$$\begin{aligned} V &= s^3 \\ &= (30)^3 \\ &= 27\ 000 \end{aligned}$$

The maximum volume of the square-based prism is 27 000 cm^3.

Key Concepts

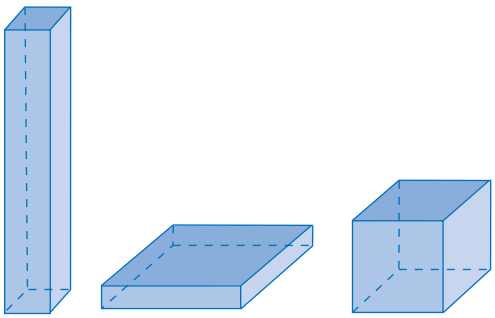
- For a square-based prism with a given surface area, a base length and a height exist that result in the maximum volume.
- The maximum volume for a given surface area of a square-based prism always occurs when the prism is a cube.
- The surface area of a cube is given by the formula $SA = 6s^2$, where s is the side length of the cube. When you are given the surface area, solve for s to find the dimensions of the square-based prism with maximum volume.

Communicate Your Understanding

C1 Describe a situation where it would be necessary to find the maximum volume of a square-based prism, given its surface area.

C2 These three boxes all have the same surface area. Which box has the greatest volume? Explain how you know.

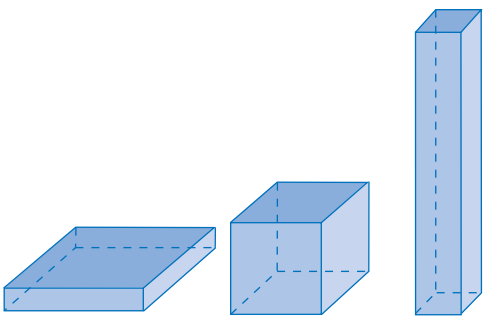
Box A Box B Box C



Practise

1. The three square-based prisms have the same surface area. Rank the prisms in order of volume from greatest to least.

Box A Box B Box C



For help with questions 2 and 3, see the Example.

2. Determine the dimensions of the square-based prism with maximum volume for each surface area. Round the dimensions to the nearest tenth of a unit when necessary.

a) 150 cm^2

b) 2400 m^2

c) 750 cm^2

d) 1200 m^2

3. Determine the volume of each prism in question 2, to the nearest cubic unit.

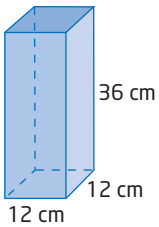
Connect and Apply

4. Use a table or a spreadsheet to conduct an investigation to find the dimensions of the square-based prism box with maximum volume that can be made with 700 cm^2 of cardboard.

5. a) Determine the surface area and the volume of the square-based prism box shown.

b) Determine the dimensions of a square-based prism box with the same surface area but with maximum volume. Round the dimensions to the nearest tenth of a centimetre.

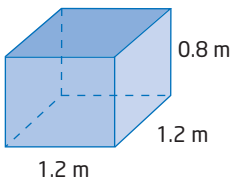
c) Calculate the volume of the box in part b) to verify that it is greater than the volume of the box in part a).



6. a) Determine the surface area and the volume of the square-based prism.

b) Determine the dimensions of a square-based prism with the same surface area but with maximum volume. Round the dimensions to the nearest tenth of a metre.

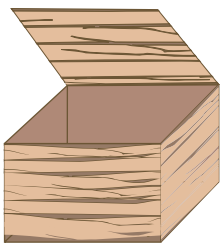
c) Calculate the volume of the prism in part b) to verify that it is greater than the volume of the original square-based prism.



7. Gurjit is building a square-based prism storage bin with a lid to hold swimming pool toys and equipment on her deck. She has 12 m^2 of plywood available.

a) Determine the dimensions of the bin with maximum volume, to the nearest tenth of a metre.

b) Determine the volume of Gurjit's bin, to the nearest cubic metre.



8. Chapter Problem Talia is packaging a DVD drive to be shipped to one of her customers. She has 2500 cm^2 of cardboard and will put shredded paper around the drive to protect it during shipping.

a) What are the dimensions of the square-based prism box with maximum volume? Round the dimensions to the nearest tenth of a centimetre.

b) What is the volume of this box?

c) If the DVD drive measures 14 cm by 20 cm by 2.5 cm, how much empty space will there be in the box?

d) What assumptions have you made in solving this problem?

Did You Know?

A DVD may have one or two sides, and one or two layers of data per side. The number of sides and layers determines the disc capacity.

Achievement Check

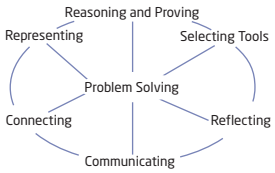
9. Kayla has 1.5 m^2 of sheet metal to build a storage box for firewood.

a) What is the surface area of the metal, in square centimetres?

b) What are the dimensions of the square-based prism box with maximum volume, including a lid?

c) If the box does not have a lid, what are the dimensions of the square-based prism box with maximum volume? Round the dimensions to the nearest tenth of a centimetre. (Hint: Make a table of possible boxes.)

d) What assumptions have you made in solving this problem?



Extend

10. Dylan has a piece of plywood that measures 120 cm by 240 cm. He wants to construct a square-based prism box to hold his sports equipment in the garage. Dylan wants to maximize the volume of the box and to keep the waste of plywood to a minimum.

a) Determine the dimensions of the box with maximum volume that he can construct, including a lid. Round to the nearest tenth of a centimetre.

b) Draw a scale diagram on grid paper to show how Dylan should cut the plywood.

c) Describe any assumptions you have made in solving this problem.

11. Sonia has a piece of stained glass that measures 20 cm by 30 cm. She is cutting the glass to make a small square-based prism box for jewellery. Sonia wants each face of the box to be made from one piece of glass.

a) Draw a scale diagram on grid paper to show how Sonia should cut the stained glass for a box with a lid.

b) Calculate the volume of this box.

c) Draw a similar scale diagram for a lidless box, showing how the glass will be cut.

d) Calculate the volume of this box.

e) Describe any assumptions you have made.