UNIT REVIEW

1. Simplify each expression (expression each as a power with positive exponents).

a) $\dfrac{(3^{-2})(3^{3})}{3^{-1}}$

$= \dfrac{3^{-2+3}}{3^{-1}}$

$= \dfrac{3^{1}}{3^{-1}}$

$= 3^{1-(-1)}$

$= 3^{2}$

b) $\dfrac{(-3)^{4}\times(-3)^{5}}{\left[(-3)^{3}\right]^{4}}$

$= \dfrac{(-3)^{4+5}}{(-3)^{3\cdot 4}}$

$= \dfrac{(-3)^{9}}{(-3)^{12}}$

$= (-3)^{9-12}$

$= (-3)^{-3}$

$= \dfrac{1}{(-3)^{3}}$

c) $\dfrac{p^{-4}q^{3}}{p^{2}q^{-2}}$

$= p^{-4-(2)}\cdot q^{3-(-2)}$

$= p^{-6}\cdot q^{5}$

$= \dfrac{q^{5}}{p^{6}}$

d) $\left(u^{2}v^{0}w^{-1}\right)^{-2}$

1

$= (u^{2})^{-2}\cdot(w^{-1})^{-2}$

$= u^{-4}\cdot w^{2}$

$= \dfrac{w^{2}}{u^{4}}$

2. Write as a root (in radical form), then evaluate [4 marks]

a) $125^{\frac{2}{3}}$

$= \left(\sqrt[3]{125}\right)^{2}$

$= 5^{2}$

$= 25$

b) $(+256)^{\frac{3}{8}}$

$= \left(\sqrt[8]{256}\right)^{3}$

$= 2^{3}$

$= 8$

3. Solve for x.

a) $10^{-3x+1} = 10^{2x-4}$

$-3x+1 = 2x-4$

$-1 \quad -1$

$-3x = 2x-5$

$-2x \quad -2x$

$\dfrac{-5x}{-5} = \dfrac{-5}{-5}$

$\boxed{x=1}$

b) $3^{3x+1} = 9^{x-2}$

$3^{3x+1} = (3^{2})^{(x-2)}$

$3^{3x+1} = 3^{2x-4}$

$3x+1 = 2x-4$

$-1 \quad -1$

$3x = 2x-5$

$-2x \quad -2x$

$\boxed{x=-5}$

c) $4^{x-3} = 8^{x+1}$

$2^{2(x-3)} = 2^{3(x+1)}$

$2x-6 = 3x+3$

$+6 \quad +6$

$2x = 3x+9$

$-3x \quad -3x$

$-1x = 9$

$\boxed{x=-9}$

a) $27^{2} = 3^{2x+1}$

$(3^{3})^{2} = 3^{2x+1}$

$3^{6} = 3^{2x+1}$

$6 = 2x+1$

$-1 \quad -1$

$5 = 2x$

$\div 2 \quad \div 2$

$\boxed{x=2.5}$

e) $3^{2x+3} = \dfrac{1}{9}$

$3^{2x+3} = 9^{-1}$

$3^{2x+3} = (3^{2})^{-1}$

$3^{2x+3} = 3^{-2}$

$2x+3 = -2$

$-3 \quad -3$

$2x = -5$

$\boxed{x=-2.5}$

4. The formula $A = P(1+i)^n$ can be used to model the growth of money when interest is compounded monthly. Solve for i.

$$i \xrightarrow{+1} \xrightarrow{\wedge n} \xrightarrow{\times P} A$$

$$A \xrightarrow{\div P} \xrightarrow{\sqrt{\ }} \xrightarrow{-1} i$$

$$\frac{A}{P} = \frac{P(1+i)^n}{P}$$

$$\sqrt[n]{\frac{A}{P}} = \sqrt[n]{(1+i)^n}$$

$$\sqrt[n]{\frac{A}{P}} - 1 = 1 + i - 1$$

$$\sqrt[n]{\frac{A}{P}} - 1 = i$$

$$\therefore\ i = \sqrt[n]{\frac{A}{P}} - 1$$

5. The volume of a sphere is given by the formula $V = \frac{4}{3}\pi r^3$. Solve for r.

$$3 \cdot V = \frac{4}{3}\pi r^3 \cdot 3$$

$$\frac{3V}{4\pi} = \frac{4\pi r^3}{4\pi}$$

$$\sqrt[3]{\frac{3V}{4\pi}} = \sqrt[3]{r^3}$$

$$\sqrt[3]{\frac{3V}{4\pi}} = r$$

$$\therefore\ r = \sqrt[3]{\frac{3V}{4\pi}}$$

6. The formula $E = mc^2$ related the mass of an object (m), the speed of light (c) and energy (E).

- Solve for m.
- Solve for c.

$$m \xrightarrow{\cdot c^2} E \qquad E \xrightarrow{\div c^2} m$$

$$\frac{E}{c^2} = \frac{m \cdot c^2}{c^2}$$

$$\frac{E}{c^2} = m$$

$$\therefore\ m = \frac{E}{c^2}$$

$$c \xrightarrow{\wedge 2} \xrightarrow{\times m} E \qquad E \xrightarrow{\div m} \xrightarrow{\sqrt{\ }} c$$

$$\frac{E}{m} = \frac{m \cdot c^2}{m}$$

$$\sqrt{\frac{E}{m}} = \sqrt{c^2}$$

$$\sqrt{\frac{E}{m}} = c$$

$$\therefore\ c = \sqrt{E/m}$$

7. Solve for x to two decimal places using a table of values to guess and check
$4^x = 300$

x	4^x
2	$4^2 = 16$
3	$4^3 = 64$
4	$4^4 = 256$
4.1	$4^{4.1} = 294$
4.11	$4^{4.11} = 298$
5	$4^5 = 1024$

x is closer to 4 than 5

$\therefore$ x is approximately 4.12.

8. Cynthia deposits money in a high interest savings account. The value of the account, V dollars, after t years is given by the equation:

$$V = 2000(1.04)^t$$

a) What does 2000 represent?
b) What does 1.04 represent?
c) How much money is the account after 13 years?
d) Cynthia will buy a used car when she has saved $5000. After how many years will Cynthia buy her car?

a) Initial amount
b) Growth factor
c) $V = 2000(1.04)^{13}$
$= 3330.15$
∴ It will grow to $3330.15
d) $\frac{5000}{2000} = \frac{2000(1.04)^t}{2000}$
$2.5 = (1.04)^t$

t	1.04^t
15	$1.04^{15} = 1.8$
20	$1.04^{20} = 2.2$
23	$1.04^{23} = 2.46$
24	$1.04^{24} = 2.56$
25	$1.04^{25} = 2.7$

∴ It'll take between 23 and 24 years.

9. Tritium, a radioactive gas that builds up in CANDU nuclear reactors, is collected, stored in pressurized gas cylinders, and sold to research laboratories. Tritium decays into helium over time. Its half-life is about 12.3 years.
a) Write an equation that gives the mass of tritium remaining in a cylinder that originally contained 500 g of tritium.
b) Estimate the time it takes until less than 5 g of tritium is present.

$A = A_0(0.5)^{t/h}$
$A_0 = 500g$, $h = 12.3$

a) $A = 500(0.5)^{t/12.3}$

b) $\frac{5}{500} = \frac{500(0.5)^{t/12.3}}{500}$

Step 1
$0.01 = 0.5^{\frac{t}{12.3}}$ Let this be "n"

Step 2

n	0.5^n
2	$0.5^2 = 0.25$
3	$0.5^3 = 0.125$
4	$0.5^4 = 0.0625$
5	$0.5^5 = 0.03125$
6	$0.5^6 = 0.015625$
6.5	$0.5^{6.5} = 0.011$
6.6	$0.5^{6.6} = 0.010$

Step 3
$\frac{t}{12.3} = n$
$12.3 \times \frac{t}{12.3} = 6.6 \times 12.3$
$t \doteq 81.2$
∴ It will take approximately 81.2 years.

10. A colony of bacteria doubles in size every 20 min. How long will it take for a colony of 20 bacteria to grow to a population of 10000?

Step 1 $A = A_0(2)^{t/d}$
$\frac{10000}{20} = \frac{20(2)^{t/20}}{20}$
$500 = 2^{t/20}$

n is approx 8.96
Step 2 $20 \cdot \frac{t}{20} = 8.96 \cdot 20$
$t = 179.2$
∴ It will take app. 179.2 min.

PRACTICE:
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