500/100

1. Solve $x^2 - 11x + 18 = 0$ using the method of factoring.

$(x-2)(x-9)=0$

$x-2=0$ $x-9=0$ $\therefore x=2$

$x=2$ $x=9$ $x=9$

2. Solve $5x^2 - 30x + 20 = 0$ using the quadratic formula. Leave your answers in exact radical form.

$x_{1,2} = \frac{-b \mp \sqrt{b^2-4ac}}{2a} = \frac{30 \mp \sqrt{900-4(5)(20)}}{10} = \frac{30 \mp \sqrt{900-400}}{10} = \frac{30 \mp \sqrt{500}}{10}$

$= \frac{30 \mp 10\sqrt{5}}{10} = \frac{\cancel{10}(3 \mp \sqrt{5})}{\cancel{10}}$ $x_1 = 3+\sqrt{5}$

$x_2 = 3-\sqrt{5}$

3. a) Write the function $f(x) = 2x^2 - 12x + 12$ in vertex form.

$=2(x^2-6x)+12$ $\frac{-6}{2}=-3$ $(-3)^2=9$

$=2(x^2-6x+9-9)+12$

$=2(x^2-6x+9)-18+12$

$=2(x-3)^2-6$

b) Starting with the vertex form from part (a) above determine the x-intercepts of the function using the method of isolating the variable. Approximate answers to two decimal places of accuracy.

$0=2(x-3)^2-6$

3 $\frac{\cancel{6}}{\cancel{2}} = \frac{\cancel{2}(x-3)^2}{\cancel{2}}$

$\sqrt{3} = \sqrt{(x-3)^2}$

$\mp\sqrt{3} = (x-3)$

$x-3=\sqrt{3}$ and $x-3=-\sqrt{3}$

$\underline{x=\sqrt{3}+3}$ $\underline{x=-\sqrt{3}+3}$

4. Determine the value of the discriminant and state the number of x-intercepts for each parabola.

a) $f(x) = 2x^2 - 8x + 19$

$D=b^2-4ac$

$=(-8)^2-4(2)(19)$

$=64-152$

$=-88$ NO X-int

b) $f(x) = -x^2 + 6x - 9$

$D=(6)^2-4(-1)(-9)$

$=36-36$

$=0$

1 x-int

5. Determine the equation, in factored form, of the quadratic function with x-intercepts $x = -7$ and $x = 4$ passing through the point (3, -20).

$y=a(x-r)(x-s)$ $r=-3$ $s=4$ $x=3$ $y=-20$

$-20=a(3-(-7))(3-4)$ $y=2(x+7)(x-4)$ ✓

$-20=a(10)(-1)$

$-20=-10a$

$\boxed{a=2}$

6. Find the point(s) of intersection (if any) of $f(x) = 5x - 2$ and $g(x) = -5x^2 - 9x + 1$ using an algebraic method. Show all steps.

$5x - 2 = -5x^2 - 9x + 1$

$5x^2 + 14x - 3 = 0$

m	A	N
-15	14	-1, +15

$\frac{(5x-1)(5x+15)}{5} = 0$

$\frac{(5x-1)\cancel{(5)}(x+3)}{\cancel{5}} = 0$

$(5x-1)(x+3) = 0$

$x_1 = \frac{1}{5}$ $x_2 = -3$

7. The profit function for a product is given by $P(x) = -4x^2 + 28x - 40$, where x is the number of products sold. Both the number of products and the profit are **in thousands**.

a) Determine how many items must be sold for the company to break-even.

$0 = -4x^2 + 28x - 40$

$0 = -4(x^2 - 7x + 10)$

$0 = x^2 - 7x + 10$

$0 = (x-2)(x-5)$

$x = 2$ $x = 5$

$\therefore$ The company needs to sell 2000 or 5000 units to break even.

b) Determine how many items must be sold for the company to make a profit of eight thousand dollars.

$8 = -4x^2 + 28x - 40$

$4x^2 - 28x + 48 = 0$

$4(x^2 - 7x + 12) = 0$

$x^2 - 7x + 12 = 0$

$(x-3)(x-4) = 0$

$x_1 = 3$

$x_2 = 4$

$\therefore$ The company needs to sell 3000 or 4000 units to make \$8000 in profit.

8. A company's profit, in thousands of dollars, on sales of video games is modeled by the function $P(n) = -2(n - 2.5)^2 + 48$, where n is the number of video games sold, in thousands.
Additionally, the company's profit, in thousands, on sales of movie videos is modeled by the function $P(n) = -(n-1)(n-11)$, where n is the number of movie videos sold, in thousands.
Calculate the maximum profit that the company can earn from both: video games and movie videos combined. Show all work.

$P(n) = -2(n-2.5)^2 + 48 - (n-1)(n-11)$

$= -2(n^2 - 5n + 6.25) + 48 - (n^2 - 12n + 11)$

$= -2n^2 + 10n - 12.5 + 48 - n^2 + 12n - 11$

$= -3n^2 + 22n + 24.5$

$= -3(n^2 - \frac{22}{3}n) + 24.5$ $\frac{22}{3} \times \frac{1}{2} = \frac{11}{3}$ $(\frac{11}{3})^2 = \frac{121}{9}$

$= -3(n^2 - \frac{22}{3}n + \frac{121}{9} - \frac{121}{9}) + 24.5$

$= -3(n - \frac{11}{3})^2 + \frac{121}{3} + 24.5$

$P(n) = -3(n - 3.67) + 64.83$

$\therefore$ Max profit \$64,830

9. A bike rental agency has 150 bikes. The owner determines that at a price of \$48 per week, he can rent all the bikes. For each \$2 increase in price, 4 fewer bikes get rented.
   a) Determine what rental charge will maximize the revenue.
   b) Each of the rented bikes need to be serviced in maintenance. Suppose it costs the owner \$5 per week per bike for maintenance. Determine the maximum profit?

a) $R(x) = (150-4x)(48+2x)$

$= 7200 + 300x - 192x - 8x^2$

$R(x) = -8x^2 + 108x + 7200$

$= -8(x^2 - 13.5x) + 7200$ $\quad \frac{-13.5}{2} = 6.75 \quad (6.75)^2 = 45.57$

$= -8(x^2 - 13.5x + 45.57 - 45.57) + 7200$

$= -8(x^2 - 13.5x + 45.57) + 364.50 + 7200$

$= -8(x - 6.75)^2 + 7564.50$

$P = 48 + (6.75)2 = \$61.50$

— OR —

$150 - 4x = 0$ $\quad 48 + 2x = 0$

$x = 37.5$ $\quad x = -24$

axis of sy = $\frac{37.5 - 24}{2} = 6.75$

$P = 48 + 2(6.75)$

$= \$61.50$

b) $P(x) = R(x) - C(x)$

$= -8x^2 + 108x + 7200 - 5(150 - 4x$

$= -8x^2 + 108x + 7200 - 750 + 20x$

$P(x) = -8x^2 + 128x + 6450$

$= -8(x^2 - 16x) + 6450$

$= -8(x^2 - 16x + 64 - 64) + 6450$

$= -8(x - 8)^2 + 512 + 6450$

$V(8, 6962)$

$\therefore$ max profit \$6962

10. Determine the equation, in standard form, of the quadratic function passing through the point (5, -1) with zeros $4+\sqrt{7}$ and $4-\sqrt{7}$.

$y = a(x-r)(x-s)$ $\quad r = 4+\sqrt{7}$ $\quad s = 4-\sqrt{7}$ $\quad P(5,-1)$

$-1 = a(5 - 4 - \sqrt{7})(5 - 4 + \sqrt{7})$

$-1 = a(1-\sqrt{7})(1+\sqrt{7})$

$-1 = a(1-7)$

$-1 = -6a$

$\boxed{a = \frac{1}{6}}$

$y = \frac{1}{6}(x - 4 - \sqrt{7})(x + 4 - \sqrt{7})$

$y = \frac{1}{6}(x^2 + \cancel{4x} - \sqrt{7}x - \cancel{4x} + \cancel{4\sqrt{7}} - \sqrt{7}x - \cancel{4\sqrt{7}} + 7)$

$\therefore y = \frac{1}{6}(x^2 - \sqrt{7}x + 7)$ ✓

11. Do the following questions from the textbook:
   Pg. 202: #1-10, 12, 14-18, 21-23
   Pg. 204: #1, 2, 6-9

Optional:
pg. 207: #12 [Hint: *# of seats* = 22 – *x*, Cost = 225 + 30(22 – *x*)]
pg. 209: #35 [Hint: *# of students* = 25 –2*x*, Cost = 5500 + 240(25 – 2*x*)]