Trigonometry Review

Communication in all questions must include:

- Enough steps shown to clearly demonstrate thinking
- Solutions that are neat and easy to follow
- Proper use of mathematical symbols
- Equal signs aligned
- Units used as required
- Concluding statements for all word problems
- Fractions reduced to lowest terms
- Correct rounding.

You will be given the following information:

$a^2 + b^2 = c^2$ ***SOH CAH TOA***

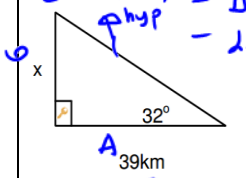
$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

$a^2 = b^2 + c^2 - 2bc\cos A$ $\cos A = \frac{b^2+c^2-a^2}{2bc}$

1. How do you know when to use SOH CAH TOA? How do you know when to use the Sine Law? How do you know when to use the Cosine Law? Describe in words and given an example.

SOH CAH TOA	- Right triangle - If given 2 sides and asked to find an angle - If given 1 side and angle and asked to find another side
SINE LAW	- Any triangle without attitude - 2 sides and one opposite angle - 2 angles and one opposite side
COSINE LAW	- Any triangle with attitude (SSS or SAS) 3 given sides and asked to find angle 2 given sides with one contained (sandwiched) angle and asked to find side

2. For each triangle below, find the missing side (x) or the missing angle (ϑ) in each diagram. (Include units, round to 1dp)



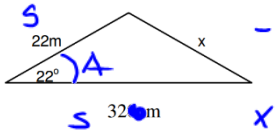
- Use SOH CAH TOA (Right tri)
- Label only given data
- with θ and A, you find T

Solution ⇒ $\tan 32 = \frac{x}{39}$ multiply B.S. by 39 to cancel out division on R.S

$39 \cdot \tan 32 = x$

$x \cong 24.4$

∴ x is approx 24.4 km



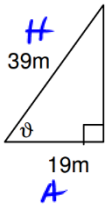
- 2 sides with a contained angle
- Cos law

$x^2 = 22^2 + 32^2 - 2 \cdot 22 \cdot 32 \cdot \cos 22$

$x^2 = 202.5251$ (√ B.S.)

$x \cong 14.2$

∴ x is 14.2 m.

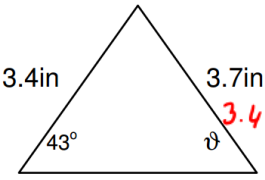


- label only given data
- CAH

$\rightarrow \cos\theta = \frac{19}{39}$ swap θ with 19/39

$\cos^{-1}(19/39) = \theta$

$\theta \cong 60.8°$



- Sine law

$3.4 \times \frac{\sin\theta}{3.4} = \frac{\sin 43}{3.7} \times 3.4$ multiply B.S. by 3.4 to cancel the division on LS

$\sin\theta = 0.6267$ swap θ with 0.6267

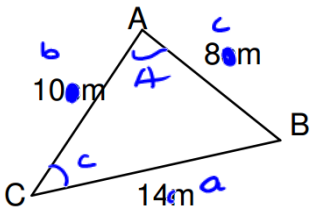
$\sin^{-1}(0.6267) = \theta$

$\theta \cong 38.8°$

3. What does it mean to "solve a triangle"?

To numerically figure out each angle and sides.

4. Solve the triangle. Summarize your answers in the chart. (Round to 1 dp)



∠A= 101.5°	a= 14m
∠B= 44.6°	b= 10m
∠C= 34°	c= 8m

$\cos A = \frac{b^2+c^2-a^2}{2\cdot b\cdot c}$

$\cos A = \frac{10^2+8^2-14^2}{2\cdot 10\cdot 8}$

$\cos A = \frac{-32}{160}$

$\cos(-32/160) = A$

$A \cong 101.5°$

$\cos B = \frac{a^2+c^2-b^2}{2\cdot a\cdot c}$

$\cos B = \frac{14^2+8^2-10^2}{2\cdot 14\cdot 8}$

$\cos B = \frac{160}{224}$

$\cos^{-1}(160/224) = B$

$B \cong 44.6$

$\cos C = \frac{a^2+b^2-c^2}{2ab}$

$\cos C = \frac{14^2+10^2-8^2}{2\cdot 14\cdot 10}$

$\cos C = \frac{232}{280}$

$\cos^{-1}(232/280) = C$

$C \cong 34°$

Q2: sin +, cos −, tan − (180°) | Q1: sin +, cos +, tan + (0°)

5. Angle A is between 0^O and 180^O. Determine all measures of angle A in each of the following cases:

a) sin A = 0.2079

sin ratio is positive in Q1 & Q2 thus, there're 2 possible angles.

$\sin^{-1} 0.2079 = A_1$ → Q1

$A_1 \cong 12°$

$A_2 = 180 - 12$

$A_2 = 168°$

check $\sin 168 = 0.2079$ ✓

$\sin 12 = 0.2079$ ✓

b) cos A = -0.8191

cos ratio is -ve in Q2 thus, there's only one angle

$\cos^{-1}(-0.8191) = A$

$A \cong 145°$

c) tan A = 1.428

tan ratio is positive in Q1, only one angle

$\tan^{-1}(1.428) = A$

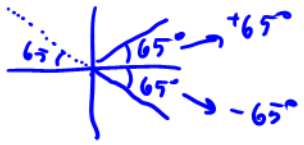
$A \cong 55°$

c) tan A = -2.145

tan is negative in Q2

$\tan^{-1}(-2.145) = A$

$A = -65°$

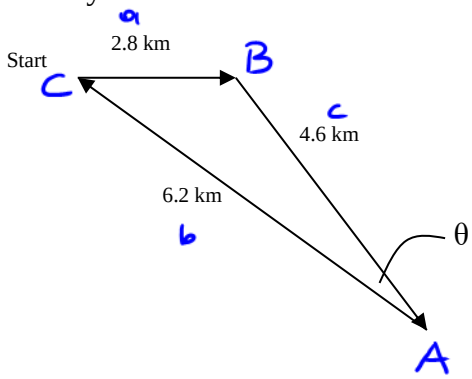


$A = 180 - 65$

$A = 115°$

check $\tan 115 = -2.145$ ✓

6. A bicycle racecourse is shown. What is the angle for the final turn, rounded to the nearest degree?



$\cos A = \frac{b^2+c^2-a^2}{2\cdot b\cdot c}$

$\cos\theta = \frac{6.2^2+4.6^2-2.8^2}{2\times 6.2\times 4.6}$ (5176)

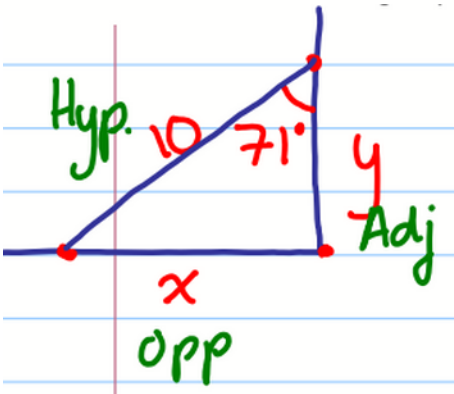
$\cos\theta = 0.9074$

$\cos^{-1}(0.9074) = \theta$

$\theta \cong 25°$

∴ The angle for the final turn is 25°

1. A ladder 10 feet long is leaning against a wall at a 71° angle. How far from the wall, is the foot of the ladder? How high up the wall does the ladder reach?



→ let x represent the distance from the wall

$\sin\theta = \frac{O}{H}$

$\sin 71° = \frac{x}{10}$

$x = 10 \sin 71°$

$= 3.3 ft$

∴ The ladder is 3.3ft from the wall

Let y represent the height of the ladder

$\cos\theta = \frac{A}{H}$

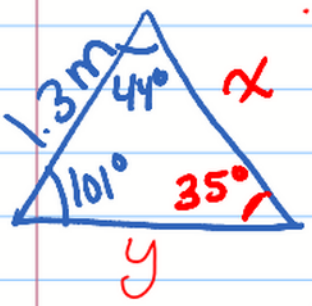
$\cos 71° = \frac{y}{10}$

$y = 10 \cos 71°$

$= 9.5 ft$

∴ The ladder is 9.5ft high.

2. Billy was making a blueprint of his home, which is triangular. One side of the triangular blueprint is 1.3 meters long. The angles in the triangle at each end of the 1.3m side are 44° and 101°. Determine the lengths of the other two sides of the blueprint.



Third angle $= 180° - 44° - 101°$

$= 35°$

Now we can use the sine law!

$\frac{x}{\sin 101°} = \frac{1.3}{\sin 35°}$

$x \sin 35° = 1.3 \sin 101°$

$x = \frac{1.3 \sin 101°}{\sin 35°}$

$= 2.2m$

$\frac{y}{\sin 44°} = \frac{1.3}{\sin 35°}$

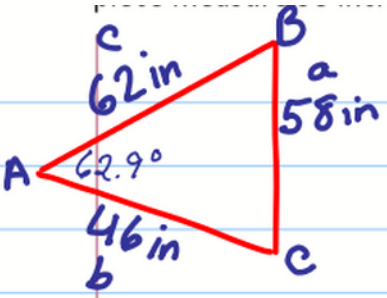
$y \sin 35° = 1.3 \sin 44°$

$y = \frac{1.3 \sin 44°}{\sin 35°}$

$= 1.6m$

∴ The other two sides are 2.2m and 1.6m long

3. A machinist is cutting out a large triangular piece of metal to make a part for a crane. The sides of the piece measure 58 inches, 46 inches, and 62 inches. What are the angles between the sides?



$\cos A = \frac{b^2+c^2-a^2}{2bc}$

$\cos A = \frac{(46)^2+(62)^2-(58)^2}{2(46)(62)}$

$\cos A = \frac{2596}{5704}$

$\cos A = 0.4551$

$A = \cos^{-1}(0.4551)$

$= 62.9°$

$\frac{\sin A}{a} = \frac{\sin B}{b}$

$\frac{\sin 62.9}{58} = \frac{\sin B}{46}$

$58 \sin B = 46 \sin 62.9$

$\sin B = \frac{46 \sin 62.9}{58}$

$\sin B = 0.7060$

$B = \sin^{-1}(0.7060)$

$= 44.9°$

$C = 180° - A - B$

$= 180° - 62.9° - 44.9°$

$= 72.2°$

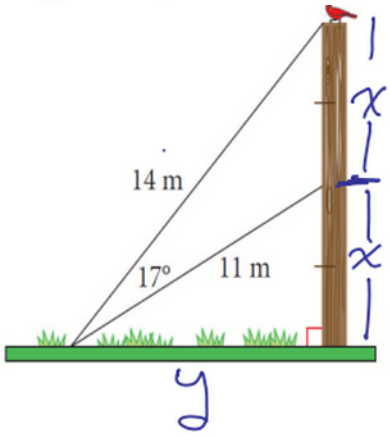
∴ The angles are 44.9°, 62.9° and 72.2°

10. A pole is supported by two guy wires, as shown. One wire is attached to the top of the pole and the other is attached at the midpoint.

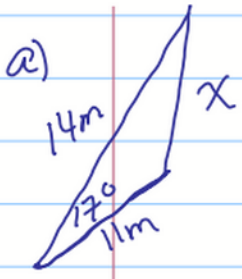
a) Determine the height of the pole.

b) How far from the base of the pole are the wires anchored?

midpoint = middle



a)



The height is 2x because the wire is attached at the midpoint.

$a^2 = b^2 + c^2 - 2bc\cos A$

$x^2 = 14^2 + 11^2 - 2(14)(11)\cos 17°$

$x^2 = 317 - 294.54$

$x^2 = 22.46$

$x = \sqrt{22.46}$

$x = 4.7\text{ m}$

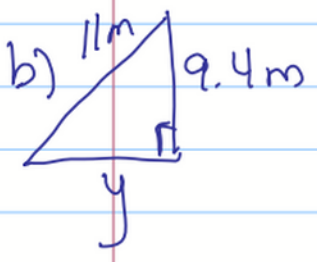
Height $= 2x$

$= 2(4.7\text{m})$

$= 9.4\text{m}$

∴ The height of the pole is 9.4 m.

b)



$a^2 + b^2 = c^2$

$y^2 + 9.4^2 = 11^2$

$y^2 = 11^2 - 9.4^2$

$y^2 = 32.64$

$y = \sqrt{32.64}$

$y = 5.7$.

∴ The wires are 5.7 m from the base of the pole.