A Dental Dilemma

The Parabola of Best Fit

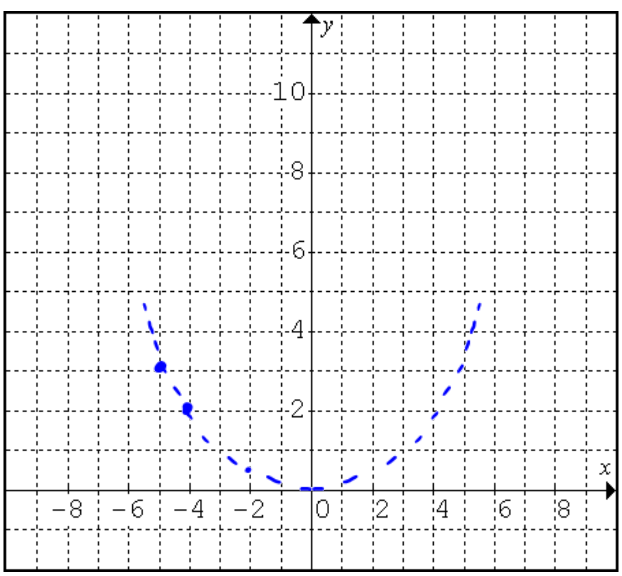
Open: Desmos App

We will be examining things in real life that are parabolic in shape. One thing would be your teeth! Dentists describe the arrangement of human teeth as a parabolic dental arcade.

1. Take a sheet of paper, and bite down on it at the corner. Be careful not to get a paper cut!!

2. Cut out the 'parabolic shape' that your teeth formed.

3. Trace your 'teeth parabola' on the grid provided. In between your front two teeth (your incisors) should be where the vertex is. The y-axis will be the axis of symmetry. Make the direction of opening of your 'teeth parabola' up.



x	y
-5	3
-4	2
-2	0.5
0	0
2	0.5
4	2
5	3

4. Locate 7 coordinates on your 'teeth parabola'. One should of course be (0, 0). Include 3 points on the left of the axis of symmetry and 3 points on the right. State the coordinates of each of these points in the table provided.

Quadratic Regression the process of modeling the relationship between the independent *x* variable and the dependent *y* variable using a quadratic relation. This process produces an equation that relates the value of *x* to the value of *y*.

- Using DESMOS, click +▾, then table. Enter all of the *x*-coordinates in the *x* column and all of the *y*-coordinates in the *y* column.
- Desmos will PLOT the points for you. Does the scatter-plot look like the one above?
- Click **Add Item** +▾, then *f(x)* expression. Type **y1**, desmos will write it as y_1, you can find **~** by clicking keypad then A B C, click ~. Type the rest of the equation and should it like this $y_1 \sim a(x_1 - h)^2 + k$ Desmos attempts to draw the best parabola (i.e. Parabola of Best Fit) through your points.

5. State the equation of your dental parabola in **vertex form** $y = a(x - h)^2 + k$. You will need to substitute the **a**, **h**, and **k** values into the equation.

$y = 0.12(x-0)^2 + 0.03$

6. The "Correlation coefficient" indicates how closely your data resembles a true parabola. A correlation coefficient of **1.0** represents a perfect parabola. What is your correlation coefficient? (It's your R^2 value)

0.999

7. How well do you think your teeth resemble a parabola? Explain.

Oil Spill

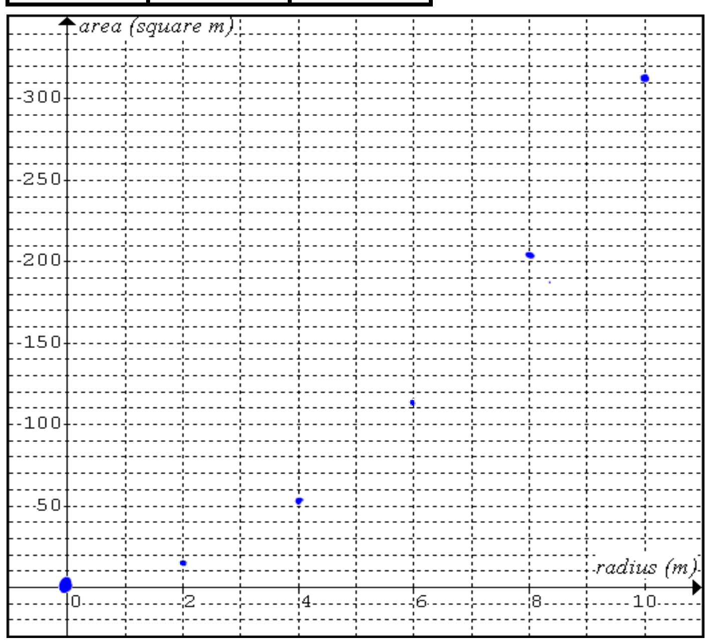
It is a calm day out at sea and the Oil Tanker Gigantica has struck an iceberg! Oil is leaking out of the ship at an extremely fast rate. The oil is spreading outwards from the ship in an approximately circular manner such that the radius of the oil spill is increasing by 2 meters every second.

Task 1: Relationship between RADIUS and AREA

1. Complete the table below, showing the dimensions of the spill for the first 5 seconds after it happened. Round your values to 2 decimal places. Recall that the formula for the area of a circle is $A = \pi r^2$. Use 3.14 for π.

Time Elapsed (s)	Radius (m)	Area of Spill (m^2)	finite differences: 1st Diff.	finite differences: 2nd Diff.
0	0	0		
1	2	12.56	12.56	25.12
2	4	50.24	37.68	25.12
3	6	113.04	62.80	25.12
4	8	200.96	87.92	25.12
5	10	314	113.04	

2. Create a scatter plot relating the radius (on the x axis) of the oil spill to area (on the y axis). This is your **graphical model**.



3. Explain THREE ways that you can tell that the relationship between the radius and the area is quadratic.
   - Line of best fit is a curve
   - 1st differences are unequal
   - 2nd differences are equal

- In DESMOS, delete the previous table and equation. Enter the radius into the x column and area into the y column. Confirm that your scatter plot matches the scatter plot in DESMOS.

4. Determine the equation:
What is the equation of the parabola? This is the **algebraic model**. What do you notice?

Let "A" represent Area (m^2)
Let "r" represent radius (m)

$y = 3.14x^2 \Rightarrow A = 3.14r^2$

Task 2: Relationship between TIME ELAPSED and RADIUS

5. What do you think the relationship would be between the time elapsed and the radius? Explain.

It's linear b/c the first differences are equal for "radius" column.

- In the desmos, enter the time elapsed into the *x* column and the radius into the *y* column. PLOT the data. Does this confirm your answer above?

yes.

6. What is the <u>**algebraic model**</u> of the relationship between the time elapsed and the radius?

   Let "t" represent time (seconds)

   Let "r" represent radius (meter)

$y=2x \rightarrow r=2t$ ____.

Task 3: Relationship between TIME ELAPSED and AREA

7. Using DESMOS, create another table 

, type the **time elapsed** values under the x column and **area** under the *y* column. You need to type this function $y_1 \sim a(x_1-h)^2+k$ to graph your table of values. **Note:** You may need to change the subscript of y and x depending on the subscript of the variables in your table of values.

8. What is the <u>**algebraic model**</u> (i.e. the equation) relating the time elapsed to area?

____.

9. How does the algebraic model for time/area compare to radius/area? Can you explain why there is such a difference?

10. What would the area of the oil spill be after one minute? Show your work.

11. If the radius increased by 3m every second instead, how would your algebraic model change?

Parabolas of Best Fit Homework

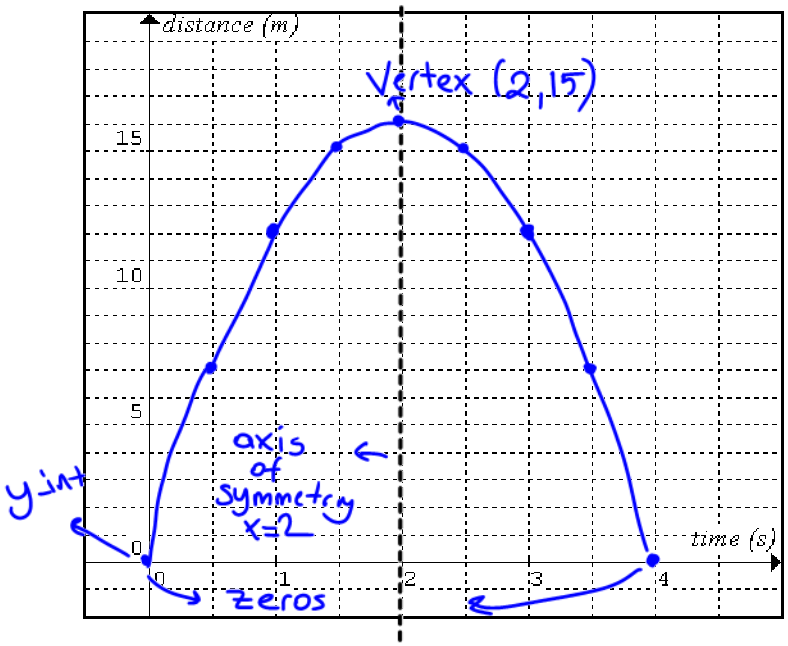
Task 1: Ron's Run

The coach of a basketball team had his players practise the following warm-up activity: Run towards the wall; touch it; then run back. To analyse their run, he placed a motion detector to record their efforts. The coach triggers the detector on as a player passes it. The following table is collected for Ron's run.

1. Complete the table of values. Is this relationship linear or quadratic? How do you know?

It's quadratic because 1^{st} diff. are unequal and 2^{nd} diff. are equal.

2. Create a scatter plot of this data.



Time Elapsed (s)	Distance from Detector (m)	finite differences: 1^{st} Diff.	finite differences: 2^{nd} Diff.
0	0		
0.5	7	7-0=7	
1	12	12-7=5	5-7=-2
1.5	15	15-12=3	3-5=-2
2	16	16-15=1	1-3=-2
2.5	15	15-16=-1	-1-1=-2
3	12	12-15=-3	-3-(-1)=-2
3.5	7	7-12=-5	-5-(-3)=-2
4	0	0-7=-7	-7-(-5)=-2

3. Does this parabola open up or down? How do the finite differences tell you?

It opens down because 2^{nd} diff are negative

4. Complete the following table. Label each item on the graph.

Point	Value	Relation to Word Problem
y-intercept	0	the distance to the motion detector.
zeros	0	when the run begins
	4	when the run ends
vertex	(2,15)	
axis of symmetry	x=2	turning point. It's where the student turns back.
max/min	15	the distance to the wall. 15m from the detector.

Task 2: A Cagey Problem

Old McDonald had a farm, EIEIO. And on that farm he had some chickens, EIEIO. He is going to cage up his chicken area, but he only has 120 metres of fencing. The chicken area is to be as large as possible and it must be completely surrounded by the fencing.

1. Complete the table. Remember: The perimeter must always add up to 120!!

2. Graph the data on the next page. Draw in the parabola of best fit.

3. What is the maximum area?

   900

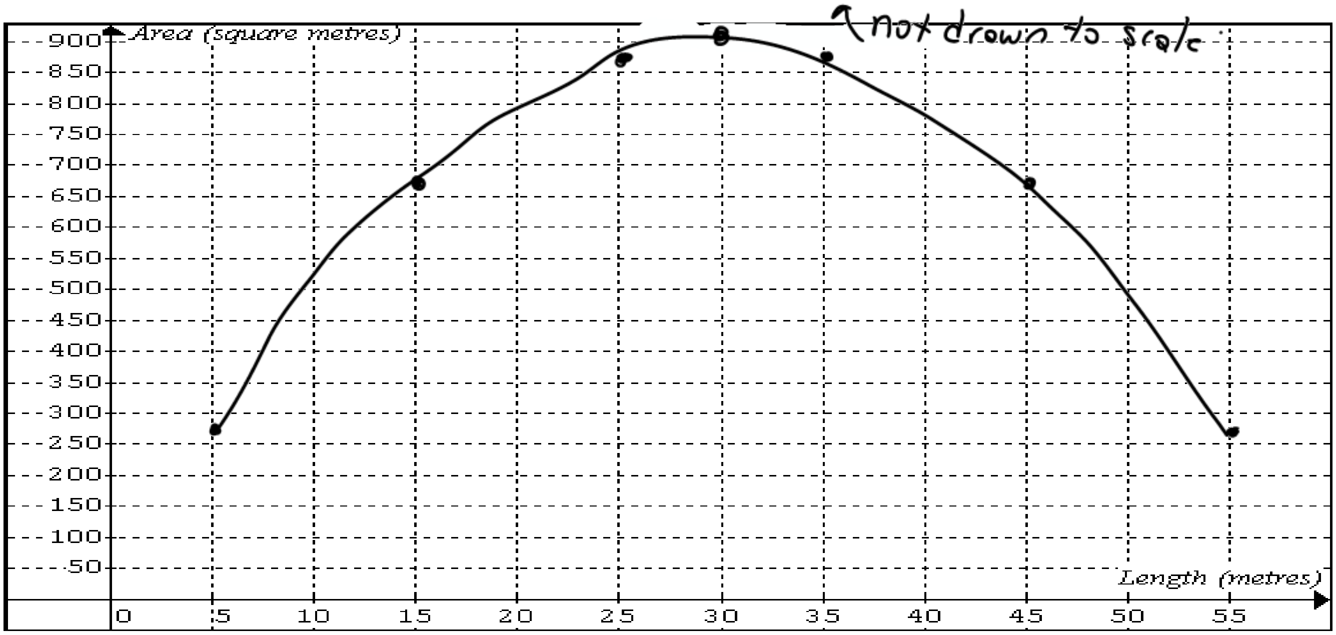
4. What do the length and width need to be to obtain that maximum area?

   30x30

5. Use the DESMOS to determine the algebraic model of the relationship between the length and area.

   $y = -(x-30)^2 + 900$

If the length of one side is... (in metres)	Diagram (not drawn to scale)	Then the width of the other side is... (make sure it all adds to 120!) (in metres)	And the area is... (length×width) (units are m²)
5	55 / 5 5 / 55	55	$55 \times 5 = 275 m^2$
15	45 / 15 15 / 45	45	$15 \times 45 = 675 m^2$
25	35 / 25 25 / 35	35	$25 \times 35 = 875 m^2$
35	25 / 35 35 / 25	25	$35 \times 25 = 875 m^2$
45	15 / 45 45 / 15	15	$45 \times 15 = 675 m^2$
55	5 / 55 55 / 5	5	$55 \times 5 = 275 m^2$



1 Old McDonald's Chicken Area

Task 3: Tickets, Anyone?

The promotions manager of a new band is deciding how much to charge for concert tickets. She has calculated that if the tickets are \$30 each, then 200 people will come to the concert. For every \$1 increase in the price, 10 less people will come. Create a table to calculate how much should be charged to MAXIMIZE the revenue from the ticket sales.

1. What is the maximum amount of money that can be earned?

   \$6250

2. What should they sell the tickets for to earn that maximum area?

   \$25

3. Use the **Desmos** to determine the algebraic model of the relationship between the ticket price and total money.

   $y = -10(x-25)^2 + 6250$

Ticket Price (\$)	Number of People	Total Money From Tickets (\$)
23	270	23×270=6210
24	260	24×260=6240
25	250	25×250 = 6250
26	240	26×240= 6240
27	230	27×230 = 6210
28	220	28×220= 6160
29	210	29×210=6090
30	200	30 × 200 = \$6000
31	190	31×190=5890
32	180	32×180= 5760

Tickets, Anyone?