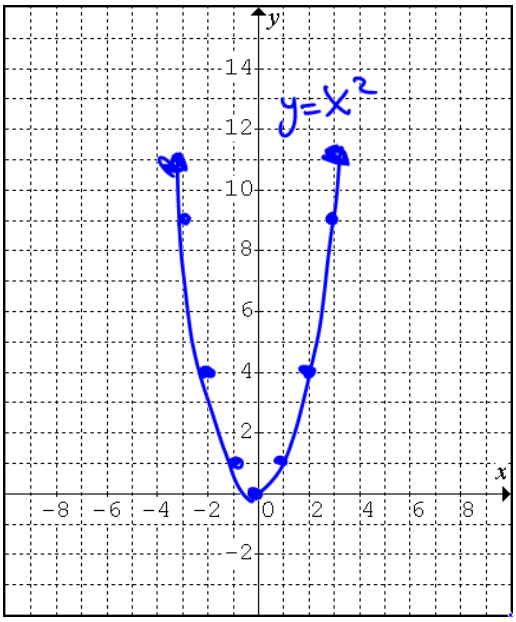
Stretches, Compressions and Reflections of Quadratic Relations

(Warm-Up) Task B: The Basic Parabola $y = x^2$

1. Complete the table of values, including the first differences.
2. Graph the parabola.

x	$y = x^2$	first differences
-4	16	
-3	9	
-2	4	
-1	1	
0	0	
1	1	1, 3, 5, 7
2	4	
3	9	
4	16	

*Recall: these are the '**step pattern**'.* ↑



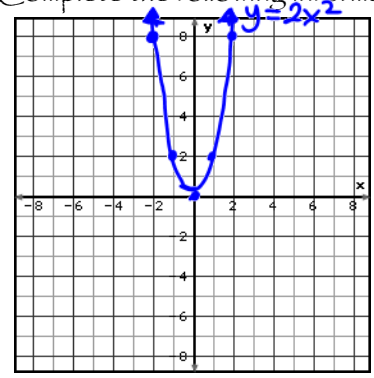
Task A: What happens when you graph $y = ax^2$?

- Change the slider for h and k to 0 and for a to 2. What equation does that produce? $y = 2x^2$

3. Describe the effect this had on the graph.

The parabola stretches vertically.

4. Complete the following information.



x	$y = 2x^2$	first differences
-3	$2(-3)^2 = 18$	
-2	$2(-2)^2 = 8$	
-1	$2(-1)^2 = 2$	
0	$2(0)^2 = 0$	2
1	$2(1)^2 = 2$	6
2	$2(2)^2 = 8$	10
3	$2(3)^2 = 18$	

vertex = (0,0)

axis of symmetry = $x = 0$

direction of opening = up

step pattern = 2 6 10

5. How does the value of a affect the step pattern?

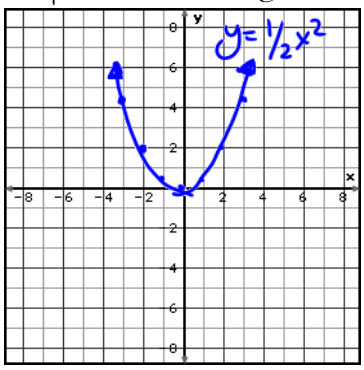
Doubled the step pattern

- Back in DESMOS, change the slider for ***a*** to ½ (or 0.5). The equation: $y = 0.5x^2$

6. Describe the effect this had on the graph.

The graph gets compressed vertically

7. Complete the following information.



x	$y = \frac{1}{2}x^2$	first differences
-3	$\frac{1}{2}(-3)^2 = 4.5$	
-2	$\frac{1}{2}(-2)^2 = 2$	
-1	$\frac{1}{2}(-1)^2 = 0.5$	
0	$\frac{1}{2}(0)^2 = 0$	0.5
1	$\frac{1}{2}(1)^2 = 0.5$	1.5
2	$\frac{1}{2}(2)^2 = 2$	2.5
3	$\frac{1}{2}(3)^2 = 4.5$	

vertex = (0,0)

axis of symmetry = x=0

direction of opening = up

step pattern = 0.5 1.5 2.5 ___

8. How does the value of ***a*** affect the step pattern?

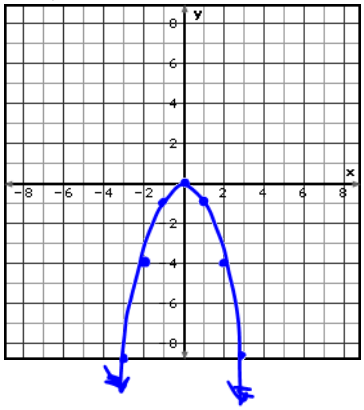
It halved the step pattern

- Back in DESMOS, change the slider for ***a*** to -1. What equation does that produce? $y = -x^2$

9. Describe the effect this had on the graph.

The graph got reflected about the "x" axis

10. Complete the following information.



x	$y = -1x^2$	first differences
-3	$-1(-3)^2 = -9$	
-2	$-1(-2)^2 = -4$	
-1	$-1(-1)^2 = -1$	
0	0	-1
1	$-1(1)^2 = -1$	-3
2	$-1(2)^2 = -4$	-5
3	$-1(3)^2 = -9$	

vertex = (0,0)

axis of symmetry = x=0

direction of opening = down

step pattern = -1 -3 -5 ___

11. How does the value of ***a*** affect the step pattern?

It negated the a values.

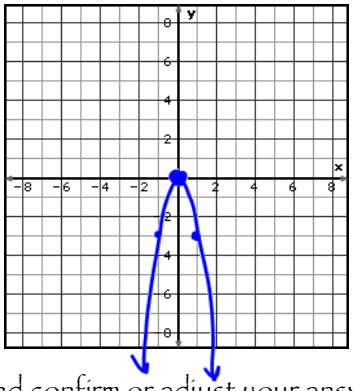
12. If the equation was $y = -3x^2$, what do you think the properties and graph would be? Try them in PENCIL!

vertex = (0,0)

axis of symmetry = x=0

direction of opening = down

step pattern = -3 -9 -15 ___



- Back in DESMOS, change the slider for ***a*** to -3 and confirm or adjust your answer above.

The graph of $y = ax^2$ produces a vertex = (0, 0)

- vertical reflection if $a < 0$ (e.g: $y = -2x^2$) axis of symmetry = x=0
- vertical stretch if $|a| > 1$ (e.g: $y = 4x^2$) step pattern = 4 12 20 28 (4·(1, 3, 5, 7))
- vertical compression if $|a| < 1$ (e.g: $y = \frac{1}{4}x^2$) direction of opening = if a>0 up, if a<0 down

Note: the | | signs around the *a* value are "absolute value" signs. It means to consider the numerical (number) value without considering the sign (positive or negative).

Task P: Practice!

13. Graph each parabola.

a. $y = 3x^2$

3(1, 3, 5)

Step Pattern: 3 9 15 ____

Vertex (0, 0)

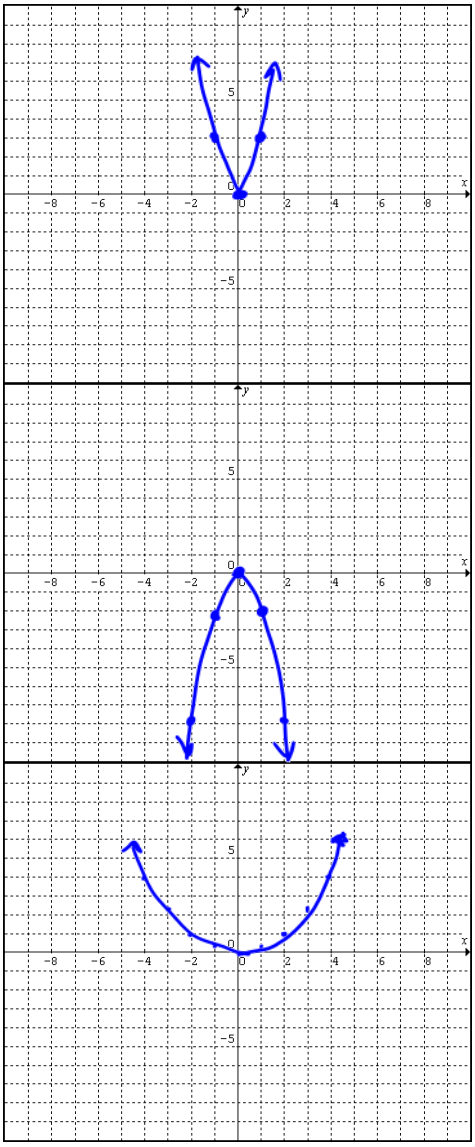
b. $y = -2x^2$

Step Pattern: -2 -6 -10 ____

c. $y = \frac{1}{4}x^2$

0.25(1, 3, 5, 7)

Step Pattern: 0.25 0.75 1.25 1.75



d. $y = 4x^2$ 4(1, 3, 5)

Step Pattern: 4 12 20 28

e. $y = -\frac{1}{2}x^2$ $-0.5(1, 3, 5, 7)$

Step Pattern: -0.5 -1.5 -2.5 -3.5

f. $y = 5x^2$ 5(1, 3, 5, 7)

Step Pattern: 5 15 25 35

g. $y = -\frac{1}{3}x^2$ $-\frac{1}{3}(1, 3, 5, 7)$

Step Pattern: $-\frac{1}{3}$ -1 $-\frac{5}{3}$ $-\frac{7}{3}$

$-1\frac{2}{3}$ $-2\frac{1}{3}$