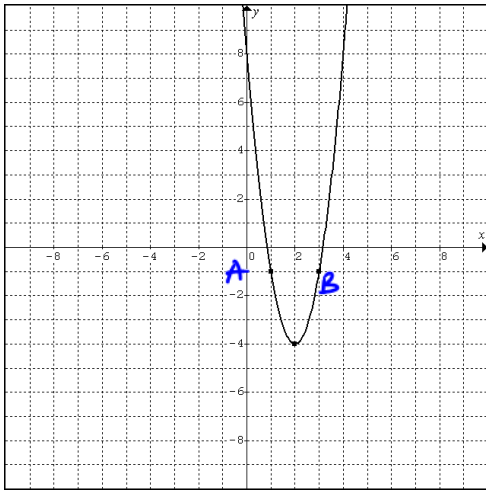


WHAT IS THE EQUATION?

Determine the equation in vertex form: $y = a(x-h)^2 + k$

h ↑ *k* ↗



The vertex is (2, -4)

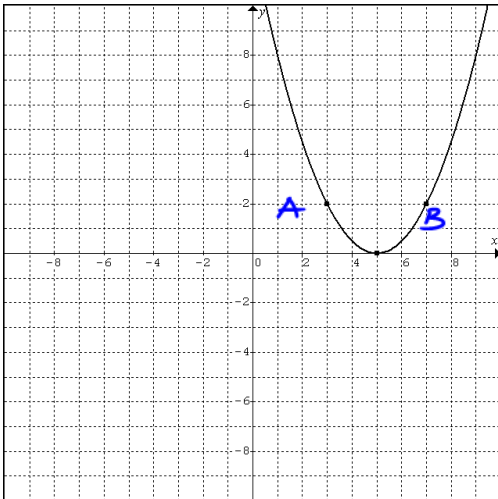
Another point is A(1, -1) find "a"
x *y*

$$\textcircled{1} y = a(x-h)^2 + k$$

$$-1 = a(1-2)^2 - 4 \quad \therefore y = 3(x-2)^2 - 4$$

$$-1 + 4 = a(-1)^2$$

$$\boxed{3 = a}$$



The vertex is (5, 0)

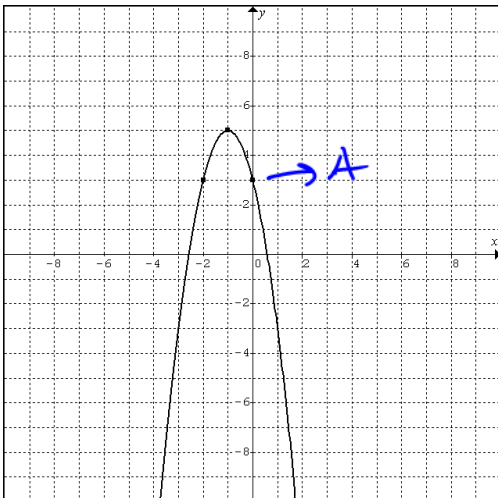
Another point is A(3, 2)

$$y = a(x-h)^2 + k$$

$$2 = a(3-5)^2 + 0 \quad \therefore y = \frac{1}{2}(x-5)^2$$

$$2 = 4a$$

$$a = \frac{1}{2}$$



The vertex is (-1, 5)

Another point is A(0, 3)

$$y = a(x-h)^2 + k$$

$$3 = a(0-(-1))^2 + 5 \quad \therefore y = -2(x+1)^2 + 5$$

$$\boxed{-2 = a}$$

The vertex is (4, -2). Another point is (7, 1).

$\begin{matrix} h & k \\ \swarrow & \searrow \\ x & y \end{matrix}$

Step 1 $y = a(x-h)^2 + k$

$$1 = a(7-4)^2 - 2$$

$$1 + 2 = a(3)^2$$

$$\frac{3}{9} = \frac{9a}{9}$$

$a = \frac{1}{3}$

$$\therefore y = \frac{1}{3}(x-4)^2 - 2$$

The vertex is (2, 0). Another point is (0, -2).

$\begin{matrix} h & k \\ \swarrow & \searrow \\ x & y \end{matrix}$

Step 1 $y = a(x-h)^2 + k$

$$-2 = a(0-2)^2 + 0$$

$$\frac{-2}{4} = \frac{4a}{4}$$

$a = -\frac{1}{2}$

$$\therefore y = -\frac{1}{2}(x-2)^2$$

The vertex is (0, -3). Another point is (1, -4).

$\begin{matrix} h & k \\ \swarrow & \searrow \\ x & y \end{matrix}$

$$y = a(x-h)^2 + k$$

$$-4 = a(1-0)^2 - 3$$

$$-4 + 3 = a$$

$a = -1$

$$\therefore y = -(x)^2 - 3$$

The vertex is (-3, -4). Another point is (-2, 1).

$\begin{matrix} h & k \\ \swarrow & \searrow \\ x & y \end{matrix}$

$$y = a(x-h)^2 + k$$

$$1 = a(-2-(-3))^2 - 4$$

$$5 = a(-2+3)^2$$

$a = 5$

$$\therefore y = 5(x+3)^2 - 4$$

The vertex is (-4, 8). Another point is (0, 0).

$\begin{matrix} h & k \\ \swarrow & \searrow \\ x & y \end{matrix}$

$$y = a(x-h)^2 + k$$

$$0 = a(0-(-4))^2 + 8$$

$$\frac{-8}{16} = \frac{16a}{16}$$

$a = -\frac{1}{2}$

$$\therefore y = -\frac{1}{2}(x+4)^2 + 8$$

The vertex is (5, 1). Another point is (1, 5).

$\begin{matrix} h & k \\ \swarrow & \searrow \\ x & y \end{matrix}$

$$y = a(x-h)^2 + k$$

$$5 = a(1-5)^2 + 1$$

$$\frac{4}{16} = \frac{16a}{16}$$

$a = \frac{1}{4}$

$$\therefore y = \frac{1}{4}(x-5)^2 + 1$$

1. Find the equation of the parabola with vertex (0,-6), opening down and a vertical compression factor of 1/3.

$h \quad k \quad a = -1$

$$y = a(x-h)^2 + k \Rightarrow \begin{matrix} h = 0 \\ k = -6 \\ a = -\frac{1}{3} \end{matrix} \quad y = -\frac{1}{3}(x)^2 - 6$$

2. Find the equation of the parabola with vertex (0,4), opening down and vertical stretch by a factor of 2.

$$\begin{matrix} h = 0 \\ k = 4 \\ a = -2 \end{matrix} \quad y = -2(x-0)^2 + 4 \Rightarrow y = -2x^2 + 4$$

3. Find the equation of the parabola compressed vertically by a factor of one-quarter, and then translated 4 units to the right and one unit up.

$$\begin{matrix} a = \frac{1}{4} \\ h = 4 \\ k = 1 \end{matrix} \quad y = \frac{1}{4}(x-4)^2 + 1$$

4. What happens to the point (3,9) on the graph of $y=x^2$ when the parabola is reflected about the x-axis then stretched vertically by a factor of two?

$$(3, 9) \longrightarrow (3, -18)$$

5. Find the value of k so that the parabola $y = -\frac{1}{3}x^2 + k$ passes through (6,8).

$$\begin{aligned} 8 &= -\frac{1}{3}(6)^2 + k \\ 8 &= -\frac{36}{3} + k \\ 8 &= -12 + k \end{aligned} \quad \boxed{k = 20}$$

6. Find the value of a and k so that the parabola passes through the points (1,-1) and (2,5)

The parabola is in the form $y = ax^2 + k$

$$\begin{aligned} \textcircled{1} \quad -1 &= a(1)^2 + k \\ \quad \downarrow \\ \quad k &= -a - 1 \\ \textcircled{2} \quad 5 &= a(2)^2 + k \\ \quad \downarrow \\ \quad 5 &= 4a + k \end{aligned} \quad \begin{aligned} \text{Sub } \textcircled{1} &\rightarrow \textcircled{2} \\ 5 &= 4a - a - 1 \\ \frac{6}{3} &= \frac{3a}{3} \\ \boxed{a = 2} & \end{aligned} \quad \begin{aligned} k &= -2 - 1 \\ \boxed{k = -3} & \end{aligned} \quad \therefore \begin{matrix} a = 2 \\ k = -3 \end{matrix}$$

7. Write an equation for the parabola with a vertex (-5,-3) passing through (-3,-11).

$$\begin{aligned} y &= a(x-h)^2 + k \\ -11 &= a(-3-(-5))^2 - 3 \\ &\quad \downarrow \\ &\quad -8 = 4a \\ &\quad \downarrow \\ &\quad \boxed{a = -2} \end{aligned} \quad \therefore y = -2(x+5)^2 - 3$$

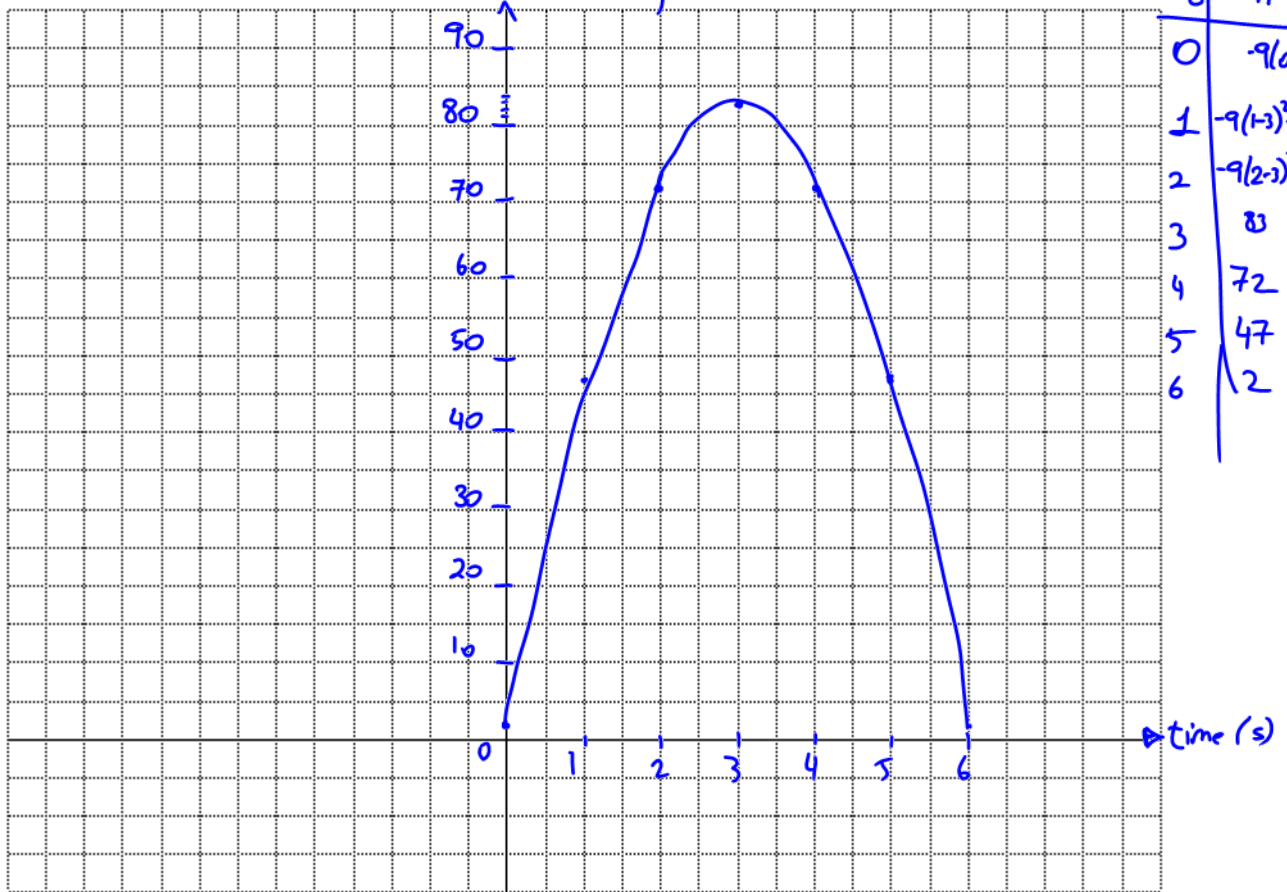
Applications of the Quadratic Relation:

8. A red flare is used by some boaters in an emergency. The flight of the flare is modelled by the function $h = -9(t - 3)^2 + 83$ where h is the height (m) of the flare and t is the time (s) that the flare is in flight.

$t = 0$
 $-81 -$

$V(3, 83)$
height (m)

a) Sketch the path of the flare.



b) What is the maximum height reached by the flare?

83 m.

c) After how many seconds does the flare reach its maximum height?

3 seconds

d) What is the height of the flare after 2 seconds?

72 m

e) Find another time that the flare is at the height in part d

4 sec.