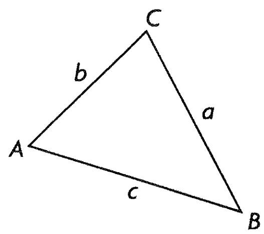
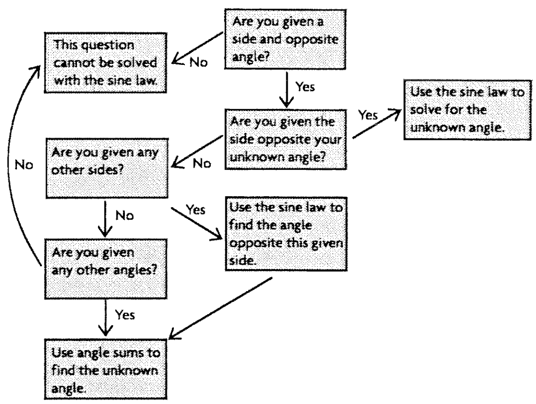
8.

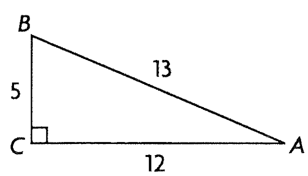


The Sine Law states, $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$.



5.1 Trigonometric Ratios of Acute Angles, pp. 280–282

1.



$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin A = \frac{a}{c}$$

$$\sin A = \frac{5}{13}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos A = \frac{b}{c}$$

$$\cos A = \frac{12}{13}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan A = \frac{a}{b}$$

$$\tan A = \frac{5}{12}$$

$$\csc A = \frac{\text{hypotenuse}}{\text{opposite}}$$

$$\csc A = \frac{c}{a}$$

$$\csc A = \frac{13}{5}$$

$$\sec A = \frac{\text{hypotenuse}}{\text{adjacent}}$$

$$\sec A = \frac{c}{a}$$

$$\sec A = \frac{13}{12}$$

$$\cot A = \frac{\text{adjacent}}{\text{opposite}}$$

$$\cot A = \frac{b}{a}$$

$$\cot A = \frac{12}{5}$$

2. $\csc\theta = \dfrac{1}{\sin\theta}$

$$\csc\theta = \frac{17}{8}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\sec\theta = \frac{17}{15}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

$$\cot\theta = \frac{15}{8}$$

3. **a)** $\csc\theta = \dfrac{1}{\sin\theta}$

$$\csc\theta = \frac{2}{1}$$

$$= 2.00$$

b) $\sec\theta = \dfrac{1}{\cos\theta}$

$$\sec\theta = \frac{4}{3}$$

$$= 1.33$$

c) $\cot\theta = \dfrac{1}{\tan\theta}$

$$\cot\theta = \frac{2}{3}$$

$$= 0.67$$

d) $\cot\theta = \dfrac{1}{\tan\theta}$

$\cot\theta = \dfrac{4}{1}$

$= 4.00$

4. a) $\cos 34 = 0.83$

b) $\sec\theta = \dfrac{1}{\cos\theta}$

$\sec 10^\circ = \dfrac{1}{\cos 10^\circ}$

$\sec 10^\circ = \dfrac{1}{0.98}$

$= 1.02$

c) $\cot\theta = \dfrac{1}{\tan\theta}$

$\cot 75^\circ = \dfrac{1}{\tan 75^\circ}$

$\cot 75^\circ = \dfrac{1}{3.73}$

$= 0.27$

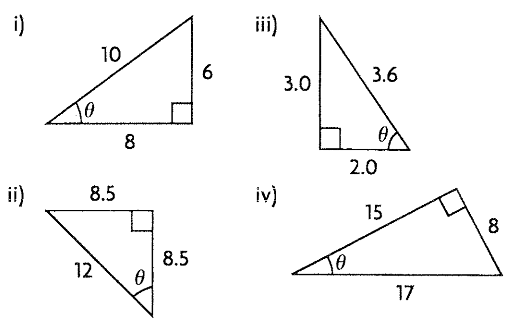
d) $\csc\theta = \dfrac{1}{\sin\theta}$

$\csc 45^\circ = \dfrac{1}{\sin 45^\circ}$

$\csc 45^\circ = \dfrac{1}{0.71}$

$= 1.41$

5.



a) $\csc\theta = \dfrac{\text{hypotenuse}}{\text{opposite}}$

$\sec\theta = \dfrac{\text{hypotenuse}}{\text{adjacent}}$

$\cot\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$

i) $\csc\theta = \dfrac{10}{6}$

$= \dfrac{5}{3}$

$\sec\theta = \dfrac{10}{8}$

$= \dfrac{5}{4}$

$\cot\theta = \dfrac{8}{6}$

$= \dfrac{4}{3}$

ii) $\csc\theta = \dfrac{12}{8.5}$

$\sec\theta = \dfrac{12}{8.5}$

$\cot\theta = \dfrac{8.5}{8.5}$

$= 1.0$

iii) $\csc\theta = \dfrac{3.6}{3.0}$

$= 1.2$

$\sec\theta = \dfrac{3.6}{2.0}$

$= 1.8$

$\cot\theta = \dfrac{2.0}{3.0}$

$= \dfrac{2}{3}$

iv) $\csc\theta = \dfrac{17}{8}$

$\sec\theta = \dfrac{17}{15}$

$\cot\theta = \dfrac{15}{8}$

b) i) From part **a**, $\csc\theta = \dfrac{5}{3}$, therefore:

$\csc\theta = 1.67$

$\sin\theta = \dfrac{1}{1.67}$

$\theta = \sin^{-1}\dfrac{1}{1.67}$

$= 37^\circ$

ii) From part **a**, $\csc\theta = \dfrac{12}{8.5}$, therefore:

$\csc\theta = 1.41$

$\sin\theta = \dfrac{1}{1.41}$

$\theta = \sin^{-1}\dfrac{1}{1.41}$

$= 45^\circ$

iii) From part **a**, $\sec\theta = \frac{3.6}{2.0}$, therefore:

$$\sec\theta = 1.8$$
$$\cos\theta = \frac{1}{1.8}$$
$$\theta = \cos^{-1}\frac{1}{1.8}$$
$$= 56°$$

iv) From part **a**, $\cot\theta = \frac{15}{8}$, therefore:

$$\cot\theta = 1.88$$
$$\tan\theta = \frac{1}{1.88}$$
$$\theta = \tan^{-1}\frac{1}{1.88}$$
$$= 28°$$

6. a) $\cot\theta = \frac{1}{\tan\theta}$

$$\cot\theta = 3.24$$
$$3.24 = \frac{1}{\tan\theta}$$
$$\tan\theta = \frac{1}{3.24}$$
$$\tan\theta = 0.31$$
$$\theta = \tan^{-1}0.31$$
$$= 17°$$

b) $\csc\theta = \frac{1}{\sin\theta}$

$$\csc\theta = 1.2711$$
$$1.2711 = \frac{1}{\sin\theta}$$
$$\sin\theta = \frac{1}{1.2711}$$
$$\sin\theta = 0.79$$
$$\theta = \sin^{-1}0.79$$
$$= 52°$$

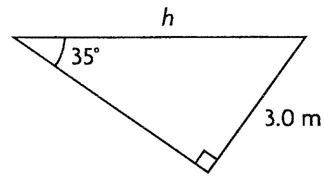
c) $\sec\theta = \frac{1}{\cos\theta}$

$$\sec\theta = 1.4536$$
$$1.4536 = \frac{1}{\cos\theta}$$
$$\cos\theta = \frac{1}{1.4536}$$
$$\cos\theta = 0.69$$
$$\theta = \cos^{-1}0.69$$
$$= 46°$$

d) $\cot\theta = \frac{1}{\tan\theta}$

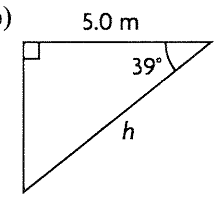
$$\cot\theta = 0.5814$$
$$0.5814 = \frac{1}{\tan\theta}$$
$$\tan\theta = \frac{1}{0.5814}$$
$$\tan\theta = 1.72$$
$$\theta = \tan^{-1}1.72$$
$$= 60°$$

7. a)



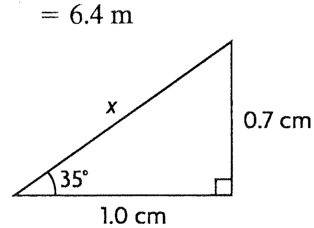
$$\sin 35° = \frac{3.0}{h}$$
$$h = \frac{3.0}{\sin 35°}$$
$$= \frac{3.0}{0.57}$$
$$= 5.2\text{ m}$$

b)



$$\cos 39° = \frac{5.0}{h}$$
$$h = \frac{5.0}{\cos 39°}$$
$$= \frac{5.0}{0.78}$$
$$= 6.4\text{ m}$$

8. a)

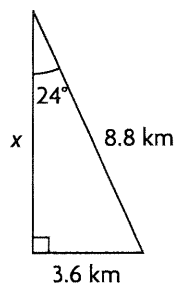


Method 1, for example:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$\sin 35° = \frac{0.7}{x}$$

$$x = \frac{0.7}{\sin 35°}$$
$$x = \frac{0.7}{0.57}$$
$$= 1.2 \text{ cm}$$
Method 2, for example:
By the Pythagorean theorem,
$x^2 = 0.7^2 + 1.0^2$
$x^2 = 0.49 + 1.00$
$x^2 = 1.49$
$x = \sqrt{1.49}$
$= 1.2$ cm

b)

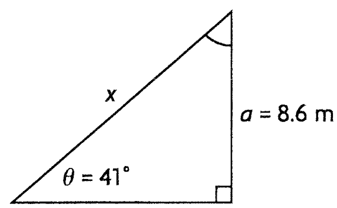


Method 1, for example:
$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$
$$\tan 24° = \frac{3.6}{x}$$
$$x = \frac{3.6}{\tan 24°}$$
$$= \frac{3.6}{0.45}$$
$$= 8.0 \text{ km}$$
Method 2, for example:
By the Pythagorean theorem,
$8.8^2 = x^2 + 3.6^2$
$x^2 = 8.8^2 - 3.6^2$
$x^2 = 77.44 - 12.96$
$x^2 = 64.48$
$x = \sqrt{64.48}$
$= 8.0$ cm

9. a) For any right triangle with acute angle θ,
$$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}}.$$
Case 1: If neither the adjacent side nor the opposite side is zero, the hypotenuse is always greater than either side and $\csc\theta > 1$.
Case 2: If the adjacent side is reduced to zero, each time you calculate $\csc\theta$, you get a smaller and smaller value until $\csc\theta = 1$.
Case 3: If the opposite side is reduced to zero, each time you calculate $\csc\theta$, you get a greater and greater value until you reach infinity. So for all possible cases in a right triangle, cosecant is always greater than or equal to 1.
b) For any right triangle with acute angle θ,
$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}.$$
Case 1: If neither the adjacent side nor the opposite side is zero, the hypotenuse is always greater than either side and $\cos\theta < 1$.
Case 2: If the opposite side is reduced to zero, each time you calculate $\cos\theta$, you get a greater and greater value until $\cos\theta = 1$.
Case 3: If the adjacent side is reduced to zero, each time you calculate $\cos\theta$, you get a smaller and smaller value until $\cos\theta = 0$. So for all possible cases in a right triangle, cosine is always less than or equal to 1.

10. $\theta = 45°$ and adjacent side = opposite side

11.

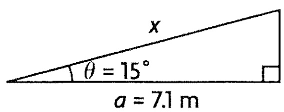


The kite, string, and ground form a right triangle. The length of the string is the hypotenuse of the right triangle and the height above ground the opposite side of the triangle, therefore:

a) $$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$\sin 41° = \frac{8.6}{x}$$
$$x = \frac{8.6}{\sin 41°}$$
$$= \frac{8.6}{0.65}$$
$$= 13.1 \text{ m}$$
b) $$\csc\theta = \frac{1}{\sin\theta}$$
$$\csc\theta = \frac{x}{8.6}$$
$$\csc 41° = \frac{1}{\sin 41°}$$

$$\frac{1}{\sin 41^\circ} = \frac{x}{8.6}$$
$$\frac{1}{0.66} = \frac{x}{8.6}$$
$$x = \frac{8.6}{0.66}$$
$$= 13.1 \text{ m}$$

12.



The wheelchair ramp and the ground form a right triangle. The length of the ramp is the hypotenuse of the right triangle and the distance from the door is the adjacent side of the triangle, therefore:

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$\cos 15^\circ = \frac{7.1}{x}$$
$$x = \frac{7.1}{\cos 15^\circ}$$
$$= \frac{7.1}{0.97}$$
$$= 7.36 \text{ m}$$

13. a) $\sec A = \dfrac{1}{\cos A}$

$$\sec A = 1.7105$$
$$\cos A = \frac{1}{1.7105}$$
$$A = \cos^{-1} 0.5846$$
$$= 54^\circ$$

b) $\cos A = 0.7512$

$$A = \cos^{-1} 0.7512$$
$$= 41^\circ$$

c) $\csc A = \dfrac{1}{\sin A}$

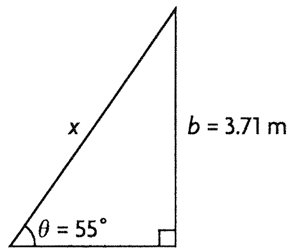
$$\csc A = 2.2703$$
$$\sin A = \frac{1}{2.2703}$$
$$A = \sin^{-1} 0.4405$$
$$= 26^\circ$$

d) $\sin A = 0.1515$

$$A = \sin^{-1} 0.1515$$
$$= 9^\circ$$

a Since b) has its angle A closest to 45°, it will have the greatest area of all the triangles (a 45°–45°–90° right triangle would have the largest possible area).

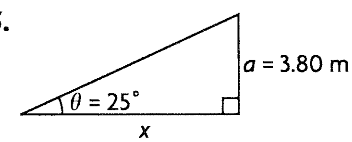
14.



The TV antenna, guy wire, and ground form a right triangle. The length of the guy wire is the hypotenuse of the right triangle and the height that the guy wire is attached is the opposite side of the triangle, therefore:

$$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}}$$
$$\csc\theta = \frac{x}{3.71}$$
$$\csc 55^\circ = \frac{1}{\sin 55^\circ}$$
$$\frac{1}{\sin 55^\circ} = \frac{x}{3.71}$$
$$\frac{1}{0.82} = \frac{x}{3.71}$$
$$x = \frac{3.71}{0.82}$$
$$= 4.5 \text{ m}$$

15.

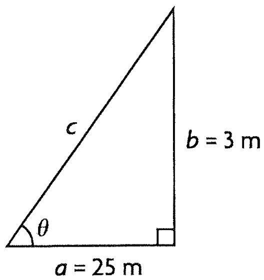


Julie and the flagpole form a right triangle from Julies head, horizontally to the flag pole, and the tip of the flag pole to Julies head. If the angle from the top of Julies head to the top of the flagpole is 25°, then the opposite side of the triangle is 5.35 − 1.55 or 3.80 m. The adjacent side of the triangle is equal to the distance between Julie and the flagpole.

$$\cot\theta = \frac{\text{adjacent}}{\text{opposite}}$$
$$\cot\theta = \frac{x}{3.8}$$
$$\cot 25^\circ = \frac{1}{\tan 25^\circ}$$
$$\frac{1}{\tan 25^\circ} = \frac{x}{3.8}$$
$$\frac{1}{0.466} = \frac{x}{3.8}$$

$$x = \frac{3.8}{0.466}$$
$$= 8.15 \text{ m}$$

16.



A 12% slope has a ratio of $\frac{12}{100}$ and can be represented at a right triangle with one side of 12 and one side 100. A similar triangle with sides 3 and 25, respectively, would have the same angles.

a) E.g., 10°

b) $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$

$$= \tan\theta = \frac{b}{a}$$

$$\tan\theta = \frac{3}{25}$$

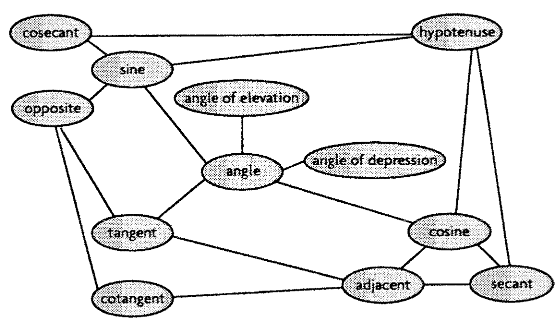
$$\theta = \tan^{-1} 0.12$$
$$= 7°$$

c) By the Pythagorean theorem, $c^2 = a^2 + b^2$.

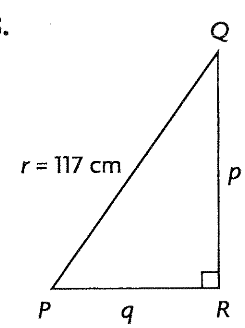
$$c^2 = 25^2 + 3^2$$
$$x^2 = 625 + 9$$
$$x^2 = 634$$
$$x = \sqrt{634}$$

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$\sin\theta = \frac{a}{c}$$
$$\sin\theta = \frac{3}{\sqrt{634}}$$
$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$\cos\theta = \frac{b}{c}$$
$$\cos\theta = \frac{25}{\sqrt{634}}$$
$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$
$$\tan\theta = \frac{a}{b}$$
$$\tan\theta = \frac{3}{25}$$
$$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}}$$
$$\csc\theta = \frac{c}{a}$$
$$\csc\theta = \frac{\sqrt{634}}{3}$$
$$\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$$
$$\sec\theta = \frac{c}{a}$$
$$\sec\theta = \frac{\sqrt{634}}{25}$$
$$\cot\theta = \frac{\text{adjacent}}{\text{opposite}}$$
$$\cot\theta = \frac{b}{a}$$
$$\cot\theta = \frac{25}{3}$$

17. For example:



18.



As given, $\tan P = 0.51$, therefore:

$\angle P = \tan^{-1} 0.51$

$\angle P = 27°$

Since this is a right triangle, $\angle R = 90°$ and:

$$\angle Q = 90° - \angle P$$
$$\angle Q = 90° - 27°$$
$$= 63°$$

$$\sin P = \frac{p}{r}$$

$$\sin 27° = \frac{p}{117}$$

$$p = 117 \times \sin 27°$$
$$= 53 \text{ cm}$$

$$\sin Q = \frac{q}{r}$$

$$\sin 63° = \frac{q}{117}$$

$$q = 117 \times \sin 63°$$
$$= 104 \text{ cm}$$

19. Since $\sec\theta = \dfrac{\text{hypotenuse}}{\text{adjacent}}$, the adjacent side must be the smallest side.

20. $\csc\theta = \dfrac{1}{\sin\theta}$

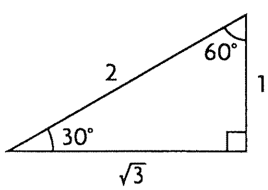
$\sec\theta = \dfrac{1}{\cos\theta}$

$\cot\theta = \dfrac{1}{\tan\theta}$

$\csc\theta$ is undefined when $\sin\theta = 0$.
$\theta = \sin^{-1} 0$
$\theta = 0°$
$\sec\theta$ is undefined when $\cos\theta = 0$.
$\theta = \cos^{-1} 0$
$\theta = 90°$
$\cot\theta$ is undefined when $\tan\theta = 0$.
$\theta = \tan^{-1} 0$
$\theta = 0°$

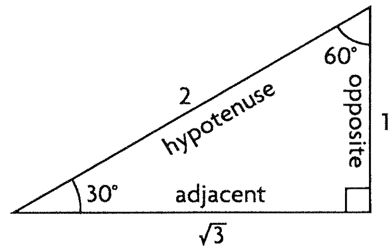
5.2 Evaluating Trigonometric Ratios for Special Angles, pp. 286–288

1. a)

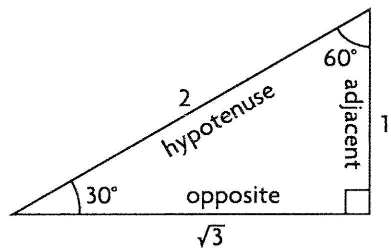


For example: The side opposite the 30° angle will be the smallest of the three sides, and is therefore the side of length 1. The hypotenuse should be the longest side, and since $\sqrt{3} < 2$, the hypotenuse is length 2. That leaves the adjacent side as length $\sqrt{3}$

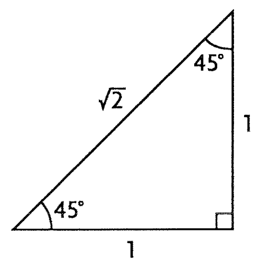
b)



c)

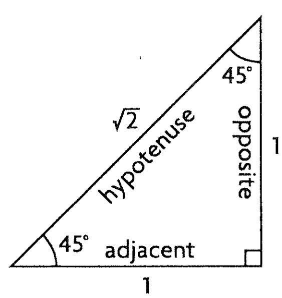


2. a)



For example: Since the triangle is a right triangle and one of the angles = 45° the other angle will also be 45°. Therefore, the two side of the right triangle will also be equal, so they are both 1. This leaves the hypotenuse as length $\sqrt{2}$.

b)



3. a) Using the special triangle 30°– 60°– 90°,

$$\sin 60° = \frac{\sqrt{3}}{2}$$

b) Using the special triangle 30°– 60°– 90°,

$$\cos 30° = \frac{\sqrt{3}}{2}$$