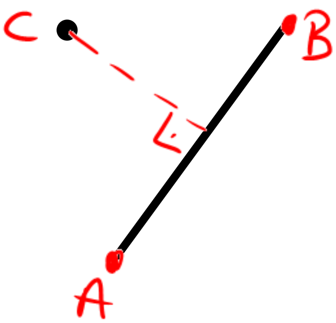
10 ACADEMIC Date ______________

Lesson 5 – Calculating the Shortest Distance from a Point to a Line

Learning Goal: I will solve problems involving the slope, midpoints, and length.
HW: p. 89 #5, 7a, 8, 18, 20, and 21

Shortest Distance from a Point to a Line



- Given the point we can draw infinite different lines to the line but…
- The shortest distance is the line that hits it at a 90^0
- The shortest distance from a point to a line is the perpendicular distance from the point to your line.

What is the shortest distance from any point to a line?

____.

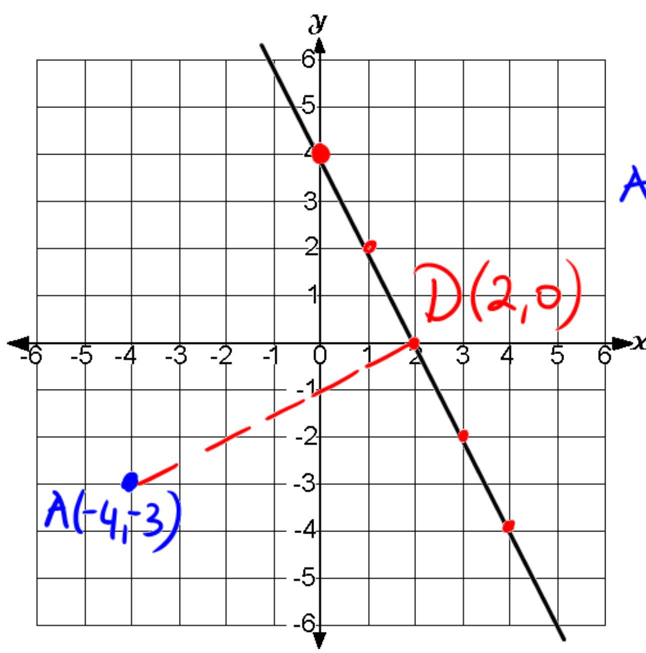
METHOD 1: FINDING THE SHORTEST DISTANCE GRAPHICALLY

Ex1 Find the shortest distance graphically between the point $(-4,-3)$ and the equation $y=-2x+4$.

① Plot the point
② Graph the equation

$y=-2x+4$

y-int



d

D(2,0)

A(-4,-3)

$d=\sqrt{(2-(-4))^2+(0-(-3))^2}$

$=\sqrt{(6)^2+(3)^2}$

$=\sqrt{45}$

$=\sqrt{9\cdot 5}$

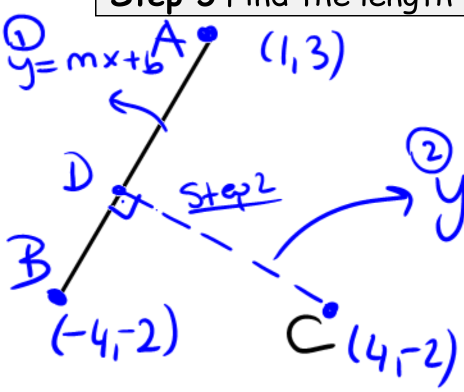
$=3\sqrt{5}$

∴ The distance is $3\sqrt{5}$ or 6.7

METHOD 2: FINDING THE SHORTEST DISTANCE ALGEBRAICALLY

Ex2 Find the shortest distance from the point $C(4,-2)$ to the line passing through the points $A(1,3)$ and $B(-4,-2)$.

Step 1 Find equation of the line AB.
Step 2 Draw a line perpendicular to AB that goes through C. Let the point on AB be called D.
Step 3 Find the equation of the line CD.
Step 4 Find D, the POI of AB and CD (substitution or elimination).
Step 5 Find the length of CD.



② $y = mx + b$

① $y = mx + b$ $\overline{AB}$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-(-2)}{1-(-4)} = \frac{5}{5} = 1$

$y = 1x + b$ $A(1,3)$

$3 = 1(1) + b$

$\boxed{b = 2}$

$\boxed{\therefore y = x + 2}$

Step 3

$m_{CD} \times m_{AB} = -1$

$m_{CD} \times 1 = -1$

$\boxed{m_{CD} = -1}$

$y = mx + b$ $m = -1$ $C(4,-2)$

$-2 = (-1)(4) + b$

$\boxed{2 = b}$

$\boxed{\therefore y = -x + 2}$

Step 4

$\left.\begin{array}{l} ①\ y = x + 2 \\ ②\ y = -x + 2 \end{array}\right\}$

$-x + 2 = x + 2$

$2 - 2 = x + x$

$0 = 2x$

$\boxed{x = 0}$

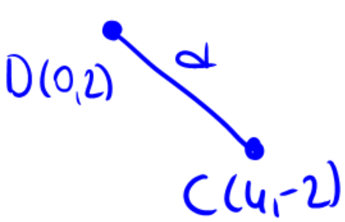
$y = x + 2$

$y = 0 + 2$

$\boxed{y = 2}$

$\therefore$ The POI is $(0,2)$

Step 5



$d = \sqrt{(4-0)^2 + (-2-2)^2}$

$= \sqrt{16 + 16}$

$= \sqrt{32} = 4\sqrt{2}$

Lesson 5 – Calculating the Shortest Distance from a Point to a Line

Ex3 Determine the shortest distance **graphically** from $A(3,-1)$ to the line $2x - y + 3 = 0$.

① Plot $A(3,-1)$

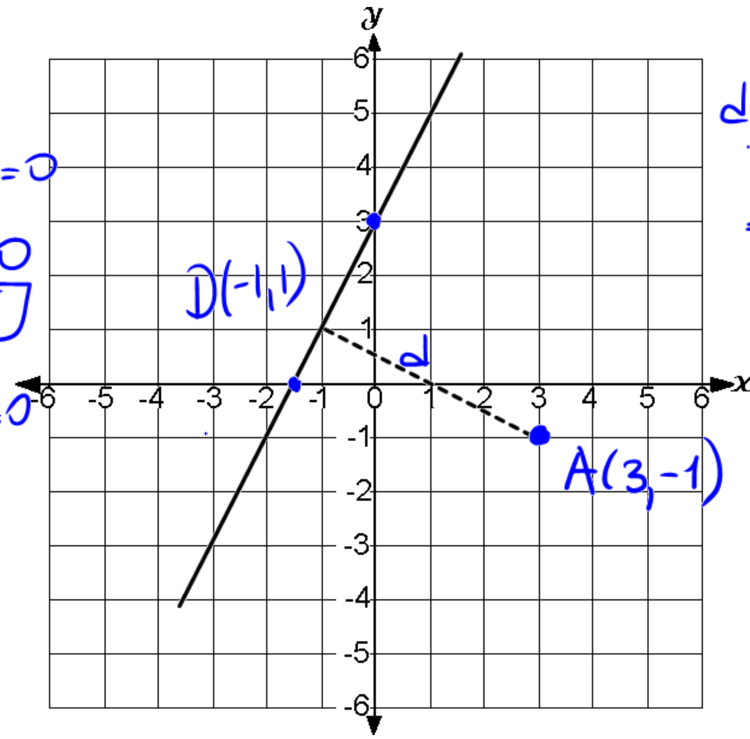
② Graph $2x-y+3=0$

x_int $2x-0+3=0$

$\boxed{x=-1.5}$

y-int $2(0)-y+3=0$

$\boxed{y=3}$



$d=\sqrt{(-1-3)^2+(1-(-1))^2}$

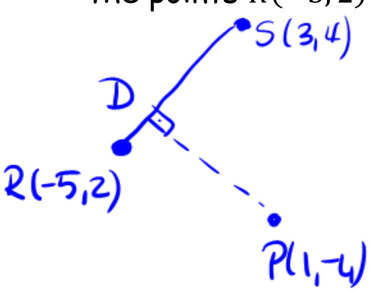
$=\sqrt{(4)^2+(2)^2}$

$=\sqrt{16+4}$

$=\sqrt{20} \longrightarrow \sqrt{4\cdot 5}$

$=2\sqrt{5}$

Ex4: Determine the shortest distance from the point $P(1,-4)$ to the line passing through the points $R(-5,2)$ *and* $S(3,4)$.



Step 1: $m_{RS}=\frac{4-2}{3-(-5)}=\frac{2}{8}=\frac{1}{4}$

$m_{RS}=\frac{1}{4}$

$y=mx+b$ $R(-5,2)$

$2=\frac{1}{4}(-5)+b$

$2+\frac{5}{4}=b \Rightarrow \boxed{b=\frac{13}{4}}$

① $y=\frac{1}{4}x+\frac{13}{4}$

$m_{PD}\times m_{RS}=-1$

$m_{PD}\times\frac{1}{4}=-1$

$\boxed{m_{PD}=-4}$

$y=mx+b$ $P(1,-4)$

$-4=-4(1)+b$

$\boxed{b=0}$

② $y=-4x$

Find the solution which is point $D(x,y)$

① $y=-\frac{1}{4}x+\frac{13}{4}$

$y=-4x$

$-4x=\frac{-x}{4}+\frac{13}{4} \Rightarrow -4x+\frac{x}{4}=\frac{13}{4} \Rightarrow \frac{\underbrace{-16x+x}_{-15x}}{1\ \cancel{4}}=\frac{13}{\cancel{4}\ 1} \Rightarrow \boxed{x=\frac{-13}{15}}$

$y=\frac{-4x}{15}=\frac{-4(13)}{15}=\frac{52}{15}$

$\therefore$ The POI $\left(\frac{-13}{15},\frac{52}{15}\right)$

Ex5: Determine the shortest distance from the point $P(-2,5)$ to the line $y = \frac{1}{2}x - 4$. Show a detailed diagram as part of your solution. Round your answer to the nearest tenth if necessary.

Plot $P(-2,5)$

$y = \frac{1}{2}x - 4$

$(2)(y+4) = \frac{x}{2}(2)$

$2y + 8 = x$

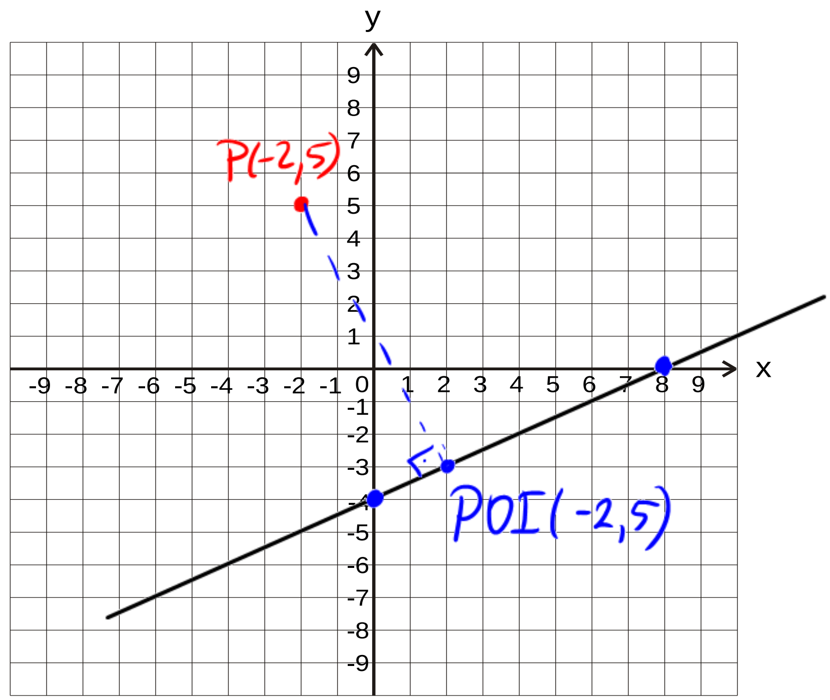
$0 = x - 2y - 8$

$x - 2y - 8 = 0$

x-int $x - 0 - 8 = 0$ $\boxed{x = 8}$

y-int $0 - 2y - 8 = 0$

$-8 = 2y \Rightarrow \boxed{y = -4}$

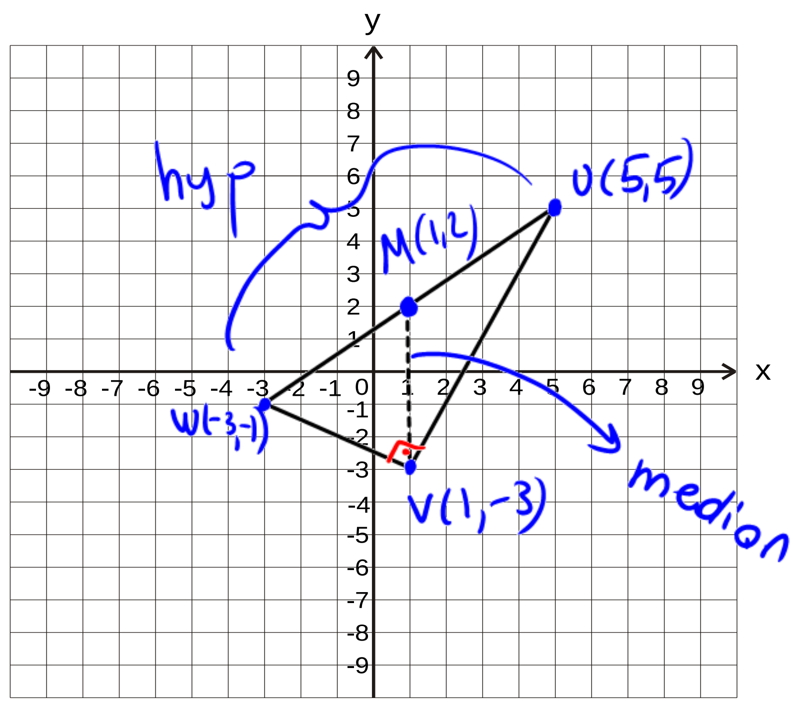


Ex6 A triangle has vertices $U(5,5), V(1,-3)$, and $W(-3,-1)$.

Verify that:

a. Triangle ΔUVW is a right triangle.

b. The median from the right angle to the hypotenuse is half as long as the hypotenuse.



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Ex 7 Triangle ABC has vertices $A(3,4)$, $B(-5,2)$, and $C(1,-4)$. Determine an equation for AE, the altitude from A to BC. What is the area of triangle ABC?

An altitude of a triangle is ____.