

Lesson 6: Verifying Properties of Triangles and Quadrilaterals

1. The line segment joining the midpoints of two sides of a triangle is parallel to the third side and half its length. Verify this property in triangle $A(1,3)$, $B(5,1)$ and $C(7,5)$. Include a labelled diagram as part of your solution.

① First find the slope of each line segment

$$m_{KL} = \frac{4-3}{4-6} = \frac{-1}{2}$$

$$m_{AB} = \frac{3-1}{1-5} = \frac{2}{-4} = \frac{-1}{2}$$

$m_{KL} = m_{AB}$

Two lines are parallel //

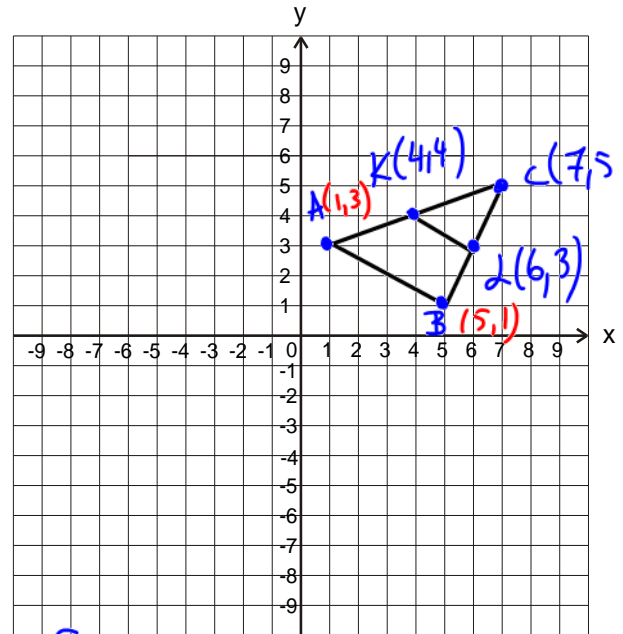
② Find the length of each line segment

$$|KL| = \sqrt{(4-6)^2 + (4-3)^2} = \sqrt{4+1} = \sqrt{5}$$

$$|AB| = \sqrt{(1-5)^2 + (3-1)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$$

$$|AB| = 2|KL|$$

∴ $|KL|$ is half of AB 's length.



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2. The midpoints of the sides of quadrilateral RSTU, with vertices $R(-4,1)$, $S(-2,5)$, $T(2,1)$, and $U(2,-7)$ are joined to form a quadrilateral. Verify that the quadrilateral formed is a parallelogram.

First calculate the length of each side

$$|KL| = \sqrt{(-3-0)^2 + (3-3)^2} = \sqrt{9} = 3$$

$$|LM| = \sqrt{(0-2)^2 + (3-(-3))^2} = \sqrt{4+36} = \sqrt{40} = 2\sqrt{10}$$

$$|MN| = \sqrt{(2-(-1))^2 + (-3-(-3))^2} = \sqrt{9} = 3$$

$$|NK| = \sqrt{(-1-(-3))^2 + (-3-3)^2} = \sqrt{4+36} = \sqrt{40} = 2\sqrt{10}$$

Then calculate the slope of each line segment

$$m_{KL} = \frac{3-3}{0-(-3)} = \frac{0}{3} = 0$$

$$m_{LM} = \frac{3-(-3)}{0-2} = \frac{6}{-2} = -3$$

$$m_{MN} = \frac{-3-(-3)}{2-(-1)} = \frac{0}{3} = 0$$

$$m_{NK} = \frac{-3-3}{-1-(-3)} = \frac{-6}{2} = -3$$

$$m_{KL} = m_{MN}$$

$$m_{LM} = m_{NK}$$

and

$$|KL| = |MN|$$

$$|LM| = |NK|$$

∴ It's parallelogram

