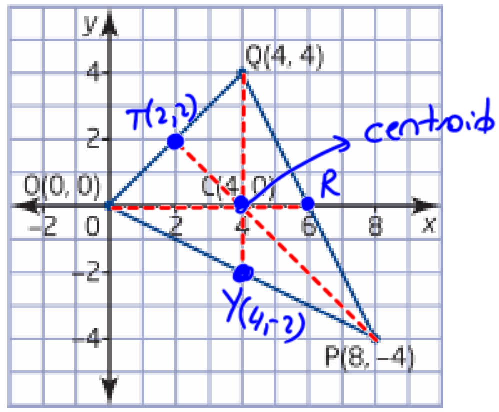
Properties of Triangles

where medians meet

Ex1. a) Verify that C(4,0) is the centroid of OPQ triangle.

b) Verify that the centroid divides each median in a 2:1 ratio.



a) $C(x,y) = \left(\frac{0+4+8}{3}, \frac{0+4+(-4)}{3}\right)$

$= (4, 0)$

b) $\overline{PC} = \sqrt{(8-4)^2+(-4-0)^2}$

$= \sqrt{16+16}$

$= \sqrt{32}$

$= 4\sqrt{2}$

$\overline{TC} = \sqrt{(4-2)^2+(0-2)^2}$

$= \sqrt{4+4}$

$= \sqrt{8}$

$= 2\sqrt{2}$

$\frac{\overline{PC}}{\overline{TC}} = \frac{4\sqrt{2}}{2\sqrt{2}} = \frac{2}{1}$

$\overline{QC} = 4-0$

$= 4$

$\overline{CY} = 0-(-2)$

$= 2$

$\frac{\overline{QC}}{\overline{CY}} = \frac{4}{2} = \frac{2}{1}$

$\overline{OC} = 4-0$

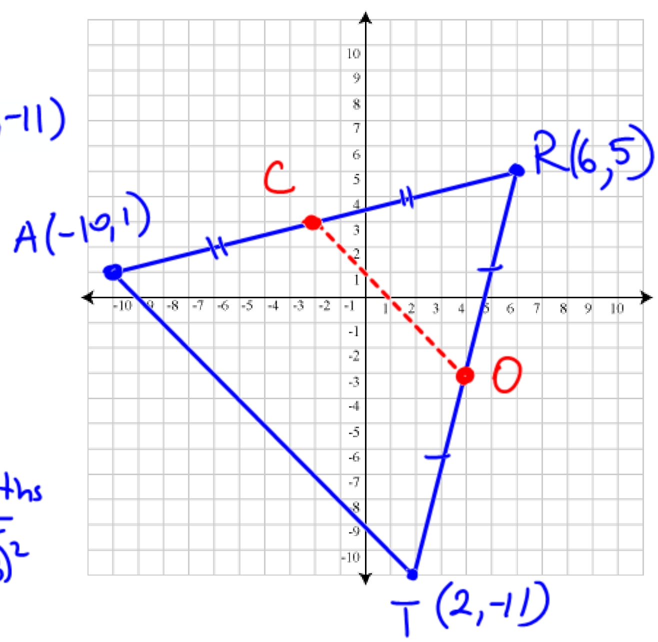
$= 4$

$\overline{CR} = 6-4$

$= 2$

$\frac{\overline{OC}}{\overline{CR}} = \frac{4}{2} = \frac{2}{1}$

Ex2. The line segment joining the midpoints of two sides of a triangle is parallel to the third side and half its length. Verify **algebraically** this property in triangle A(-10,1), R(6,5), T(2,-11). Include a labelled diagram as part of your solution.



Step1: Midpoint of $\overline{AR}$, $C(x,y)$

$C(x,y) = \left(\frac{-10+6}{2}, \frac{1+5}{2}\right) = (-2, 3)$

Step2: Midpoint of $\overline{RT}$, $O(x,y)$

$O(x,y) = \left(\frac{6+2}{2}, \frac{5+(-11)}{2}\right) = (4, -3)$

Step3: Comparison of Slopes

$m_{CO} = \frac{-3-3}{4-(-2)} = \frac{-6}{6} = -1$

$m_{AT} = \frac{-11-1}{2-(-10)} = \frac{-12}{12} = -1$

Since $m_{CO} = m_{AT}$,

$m_{CO} \parallel m_{AT}$

Step4 Compare lengths

$d_{CO} = \sqrt{(-2-4)^2+(-3-3)^2}$

$= \sqrt{36+36}$

$= \sqrt{72}$

$d_{CO} = 6\sqrt{2}$

$d_{AT} = \sqrt{(-10-2)^2+(1-(-11))^2}$

$= \sqrt{144+144}$

$= \sqrt{288}$

$d_{AT} = 12\sqrt{2}$

Step5

$\frac{d_{AT}}{d_{CO}} = \frac{12\sqrt{2}}{6\sqrt{2}} = 2$

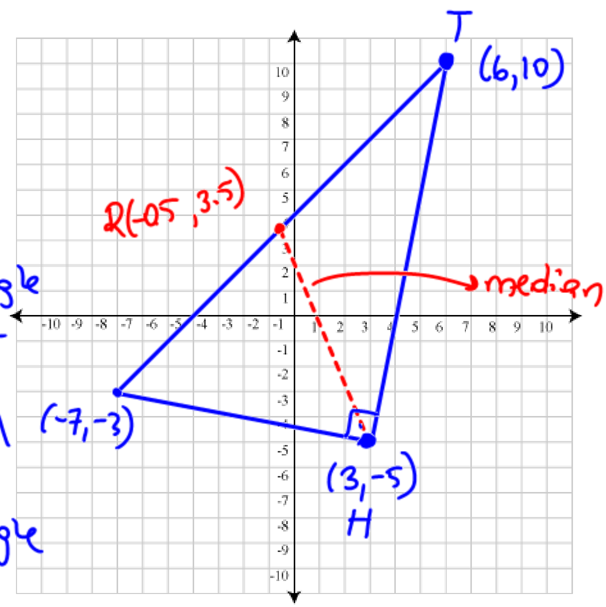
Ex3. A triangle has vertices $M(-7,-3)$, $T(6,10)$, $H(3,-5)$

i) Verify that:

a. Triangle ΔMTH is a right triangle. ✓

b. The median from the right angle to the hypotenuse is half as long as the hypotenuse. ✓

ii) Calculate the perimeter.



a) if $m_{MH} \times m_{TH} = -1$, then ΔMTH is a right triangle

$m_{MH} = \frac{-3-(-5)}{-7-3} = \frac{-3+5}{-10} = \frac{2}{-10} = -1/5$

$m_{TH} = \frac{-5-10}{3-6} = \frac{-15}{-3} = 5$

$-\frac{1}{5} \times 5 = -1$ $\therefore$ ΔMTH is a right triangle

b) Midpoint of $\overline{MT}$, $R(x,y)$

$R(x,y) = \left(\frac{-7+6}{2}, \frac{-3+10}{2}\right) = (-0.5, 3.5)$

step 1

$d_{RH} = \sqrt{(-0.5-3)^2 + (3.5-(-5))^2}$

$= \sqrt{(-3.5)^2 + (3.5+5)^2}$

$= \frac{13\sqrt{2}}{2}$

$\boxed{d_{RH} = 6.5\sqrt{2}}$

step 2

$d_{MT} = \sqrt{(-7-6)^2 + (-3-10)^2}$

$= \sqrt{(-13)^2 + (-13)^2}$

$\boxed{d_{MT} = 13\sqrt{2}}$

Step 3

$\frac{d_{MT}}{d_{RH}} = \frac{13\sqrt{2}}{6.5\sqrt{2}}$

$\frac{d_{MT}}{d_{RH}} = 2$

c) $P = d_{MT} + d_{TH} + d_{MH}$

step 1

$d_{TH} = \sqrt{(6-3)^2 + (10-(-5))^2}$

$= \sqrt{9 + 225}$

$= \sqrt{234}$

$= 3\sqrt{26}$

step 2

$d_{MH} = \sqrt{(-7-3)^2 + (-3-(-5))^2}$

$= \sqrt{100+4}$

$= \sqrt{104}$

$= 2\sqrt{26}$

Step 3 $P = 13\sqrt{2} + 3\sqrt{26} + 2\sqrt{26}$

$= 12\sqrt{2} + 5\sqrt{26}$

COMPLETE: p.89 - #5, 9 and p.125 - #13