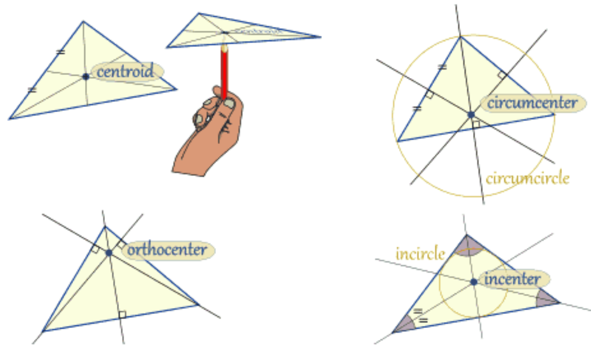
Go to http://bit.ly/trianglecentres

Where is the center of a triangle?

There are actually *thousands* of centers! Here are the 4 most popular ones:



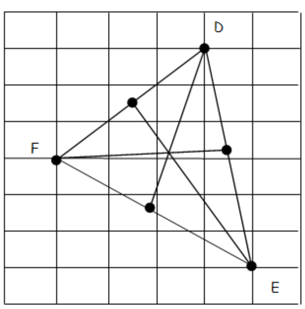
Centroid, Circumcenter, Incenter and Orthocenter

1. CENTROID

The **centroid** of a triangle is:

* The point where the three medians of the triangle intersect.
* The 'center of gravity' of the triangle .

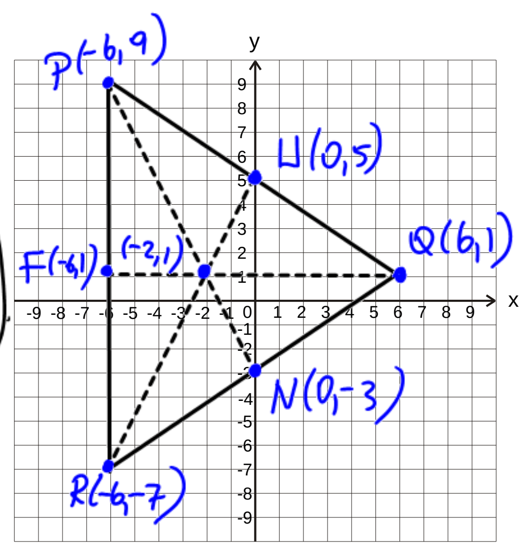
Diagram:



To determine the centroid **algebraically**:

1) Label vertices (or use letters provided)
2) find median of x and y coordinates: $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$
3) This is the *centroid* (centre of mass)

Ex1. Determine the centroid of the triangle with vertices P(-6,9) Q(6,1) and R(-6,-7) **geometrically**.



$G(x,y) = \left(\frac{-6+0+0}{3}, \frac{1+5-3}{3}\right)$

$= \left(\frac{-6}{3}, \frac{3}{3}\right)$

$G = (-2,1)$

$G(x,y) = \left(\frac{-6+(-6)+6}{3}, \frac{9+(-7)+1}{3}\right)$

OR $G(x,y) = \left(\frac{-6-6+6}{3}, \frac{9-7+1}{3}\right)$

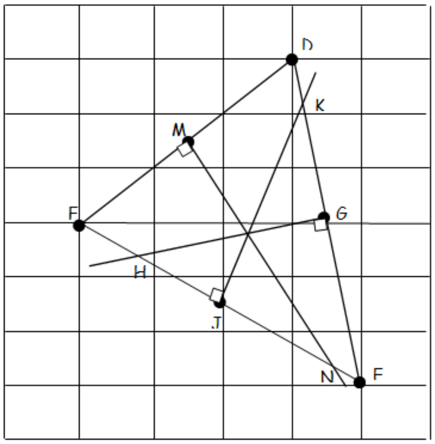
$G = (-2,1)$

2. CIRCUMCENTRE

The **circumcentre** of a triangle is

* The point where the three perpendicular bisectors of a triangle meet.

Diagram:



To determine the circumcentre **algebraically**:
1) find equations of 3 right bisectors GH, JK and MN (find midpoints G, J and M, find slopes of DE, EF, and DF, find negative reciprocal for perpendicular slopes, use G, J, and M as points on the right bisectors)
2) find point of intersection of GH and JK (solve linear system using substitution)
3) this point is the *circumcentre*
4) confirm that this point is on the third right bisector MN (using LS = RS)

Ex2. Determine the circumcentre of the triangle with vertices D(-1,3) E(3,-1) and F(-1,-5) **geometrically** and **algebraically** .

Bisector $\overline{AC}$

$m_{AC} = \frac{1-(-1)}{1-(-1)}$

$= \frac{2}{2}$

$m_{AC} = 1$ A(1,1)

$y - y_1 = m(x - x_1)$

$y - 1 = 1(x - 1)$

$y = x - 1 + 1$

$\boxed{y = x}$

Bisector EC

$m_{EC} = \frac{-1-(-1)}{3-(-1)}$

$m_{EC} = 0$ E(3,-1)

$y - y_1 = m(x - x_1)$

$y - (-1) = 0(x - 3)$

$\boxed{y = -1}$

Bisector $\overline{NC}$

$m_{NC} = \frac{-3-(-1)}{1-(-1)}$

$= \frac{-3+1}{1+1}$

$m_{NC} = \frac{-2}{2}$

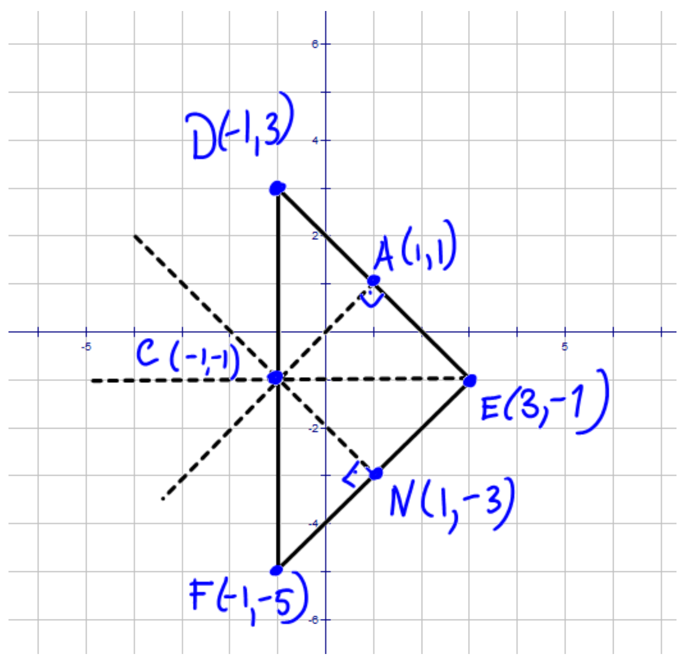
$m_{NC} = -1$ N(1,-3)

$y - y_1 = m(x - x_1)$

$y - (-3) = -1(x - 1)$

$y + 3 = -x + 1$

$\boxed{y = -x - 2}$



You can find the intersecting point of any 2 of the bisectors
let's pick $\overline{AC}$ and $\overline{NC}$

① $y = x$
② $y = -x - 2$

sub ① into ②

$x = -x - 2$

$2x = -2$

$\boxed{x = -1}$

$y = x$

$\boxed{y = -1}$

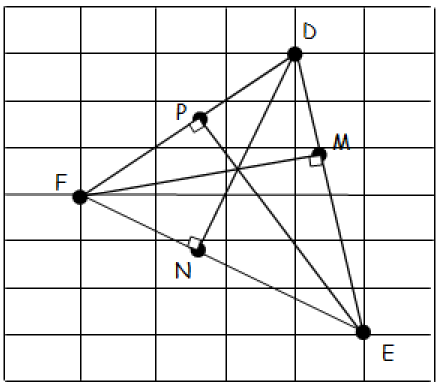
∴ POI is (-1,-1)

3. ORTHOCENTRE

The **orthocentre** of a triangle is

* The point where the three altitudes of a triangle intersect.

Diagram:



To determine the orthocentre <u>**algebraically**</u>:
1) find the equations of the 3 altitudes DN, FM, and EP (find slopes of EF, ED, & FD, and use negative reciprocals to find perpendicular slopes of DN, FM, and EP; use D, E, and F as points on the altitudes)
2) find point of intersection of DN and FM (solve linear system using substitution)
3) this point is the ***orthocentre***
4) confirm that this point is on the third altitude EP (using LS = RS)

Ex3. Determine the orthocentre of the triangle with vertices A(-2,5), B(5,1) and C(-4, -5) **geometrically** and **algebraically**.

altitude AN

$m_{CB} = \frac{-5-1}{-4-5}$

$= \frac{-6}{-9}$

$m_{CB} = \frac{2}{3}$

$m_{AN} = -\frac{3}{2}$ A(-2,5)

$y - y_1 = m(x - x_1)$

$y - 5 = -\frac{3}{2}(x+2)$

$y = -\frac{3x}{2} - 3 + 5$

① $\boxed{y = -\frac{3x}{2} + 2}$

UC

$m_{AB} = \frac{5-1}{-2-5} = \frac{4}{-7}$

$m_{AB} = -\frac{4}{7}$

$m_{UC} = \frac{7}{4}$ C(-4,-5)

$y - y_1 = m(x - x_1)$

$y + 5 = \frac{7}{4}(x+4)$

$y = \frac{7x}{4} + 7 - 5$

② $\boxed{y = \frac{7x}{4} + 2}$

FB

$m_{AC} = \frac{-5-5}{-4+2}$

$= \frac{-10}{-2}$

$m_{AC} = 5$ B(5,1)

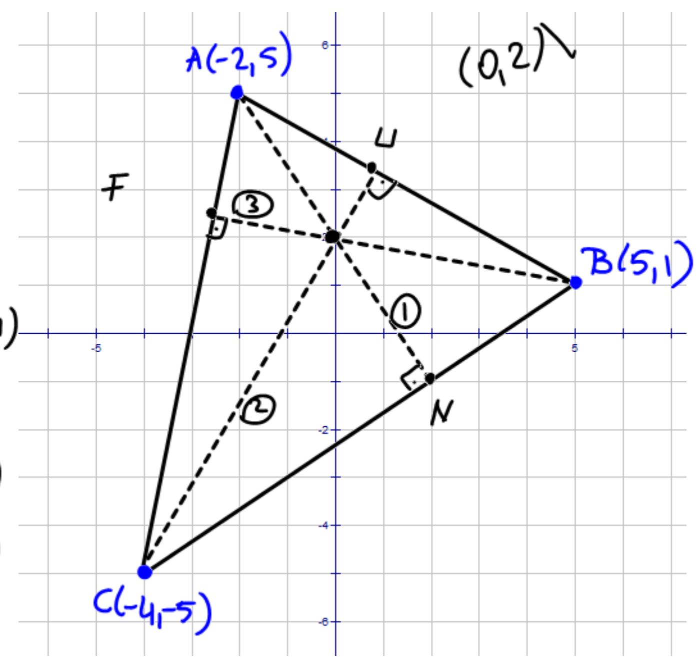
$m_{FB} = -\frac{1}{5}$

$y - y_1 = m(x - x_1)$

$y - 1 = -\frac{1}{5}(x - 5)$

$y - 1 = -\frac{x}{5} + 1$

③ $\boxed{y = -\frac{x}{5} + 2}$



① $y = -\frac{3}{2}x + 2$

② $y = \frac{7x}{2} + 2$

sub ① → ②

$-\frac{3x}{2} + 2 = \frac{7x}{2} + 2$

$-\frac{3x}{2} - \frac{7x}{2} = 0$

$-\frac{10x}{2} = 0$

$\boxed{x = 0}$

$y = \frac{7(0)}{2} + 2$

$\boxed{y = 2}$

CHECK

$y = -\frac{5x}{2} + 2$

$2 = -\frac{5(0)}{2} + 2$

$2 = 2$ ✓

SEATWORK p.91 #28 and p.99 #24abc