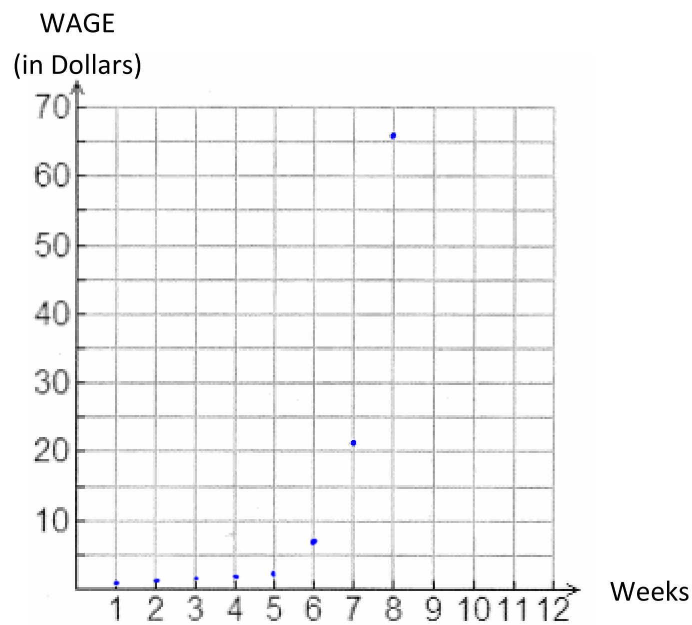
The Exponential Function

Frodo accepted to cut his uncle Bilbo Baggins' lawn for 8 weeks in the summer. Little did he know that he was going to embark on a journey to destroy the Lord of the Rings. Bilbo offered to pay him $5 Shire dollars per week, plus a $10 bonus. However, Frodo had something else in mind and proposed getting paid 3 cents the first week, 9 cents the second week, 27 cents the third week, and so on, with each subsequent week's pay being 3 times that of the previous week. If you were Bilbo, would you accept Frodo's proposal?

Method 1: $C = 5(8) + 10$
$= \$50$

Method 2:

Week Number	Wage for week (in cents)	First Differences	Second Differences
1	3		
2	9	6	
3	27	18	12
4	81	54	36
5	243	162	108
6	729	486	324
7	2187	1458	972
8	6561	4374	2916



Recall: *If the first differences are equal, it is linear; the second differences are equal it is quadratic relationship.*

Is there a pattern in the first differences?

The consecutive one is 3 times the previous.

What do you notice about the entries in the wage column?

The entries grow by 3.

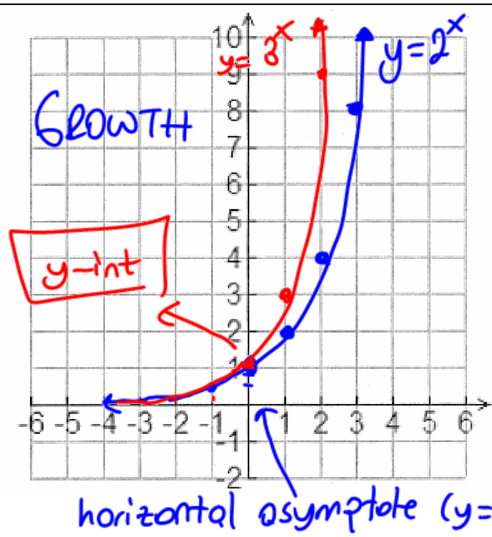
How would you express this relationship algebraically?

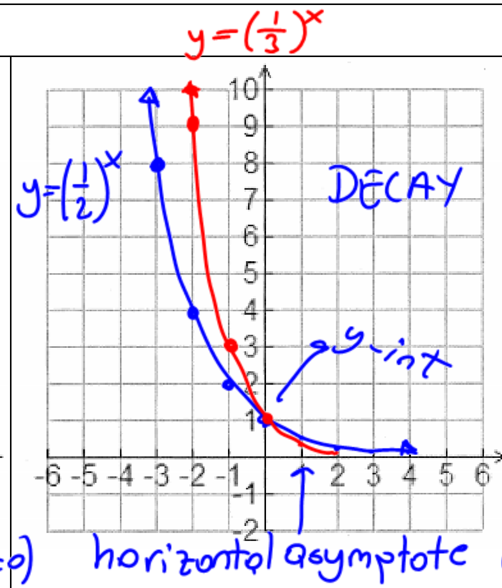
$Wage = 3^x$

1. a. Using the table of values, draw each exponential function.

x	$y = 2^x$
-3	$2^{-3} = 1/8$
-2	$2^{-2} = 1/4$
-1	$2^{-1} = 1/2$
0	$2^0 = 1$
1	$2^1 = 2$
2	$2^2 = 4$
3	$2^3 = 8$

x	$y = 3^x$
-3	$3^{-3} = 1/27$
-2	$3^{-2} = 1/9$
-1	$3^{-1} = 1/3$
0	$3^0 = 1$
1	$3^1 = 3$
2	$3^2 = 9$
3	$3^3 = 27$





x	$y = (\frac{1}{2})^x$
-3	$(\frac{1}{2})^{-3} = 8$
-2	4
-1	2
0	1
1	1/2
2	1/4
3	1/8

x	$y = (\frac{1}{3})^x$
-3	$(\frac{1}{3})^{-3} = 27$
-2	9
-1	3
0	1
1	1/3
2	1/9
3	1/27

b. What is y-intercept for each of the graphs? Label it on the plane. (0,1)

c. As the x values increase what do you notice about the y values?
As x approaches "+" infinity (+∞), y values increase.

d. As the x values decrease what do you notice about the y values?
As x approaches "−" infinity (-∞), y values decrease

d. Do you think this graph will ever intersect with y =0 line (x axis)?
No, because y is not going to be zero HORIZONTAL ASYMPTOTE

f. State the domain and range:

	$y = 2^x$	$y = 3^x$
D:	$\{x \in R\}$	$\{x \in R\}$
R:	$\{y \in R \mid y > 0\}$	$\{y \in R \mid y > 0\}$

g. What are the common characteristics of these curves?
- same domain
- same range
- Increasing graph (GROWTH)
- y-int.

b. What is y-intercept for each of the graphs? Label it on the plane. (0,1)

c. As the x values increase what do you notice about the y values?
They decrease. As you go right horizontally, y values decrease

d. As the x values decrease what do you notice about the y values?
As you go left horizontally, y values increase.

d. Do you think this graph will ever intersect with y =0 line (x axis)?
No. HORIZONTAL ASYMPTOTE y=0 line

f. State the domain and range:

	$y = (\frac{1}{2})^x$	$y = (\frac{1}{3})^x$
D:	$\{x \in R\}$	$\{x \in R\}$
R:	$\{y \in R \mid y > 0\}$	$\{y \in R \mid y > 0\}$

g. What are the common characteristics of these curves?
- same domain
- same range
- decreasing graph (DECAY)
- y-int

Notes about Exponential Functions

The exponential function $f(x) = b^x$ is to be added to our list of parent functions.

Exponential functions can be used to model population **growth** or the temperature of a liquid as it cools off.

When b > 1, the exponential function decreases to the left and increases to the right. This is called <u>exponential growth</u>.

When 0 < b < 1, the exponential function increases to the left and decreases to the right. This is called <u>exponential decay</u>.

The x-axis is called a horizontal asymptote for all 4 graphs.

The equation of this line is $y = 0$.

The domain of $f(x) = b^x$ is $\{x \in R\}$.

The range of $f(x) = b^x$ is $\{y \in R \mid y > 0\}$. y is an element of Real #s, y is greater than 0

The y-intercept of $f(x) = b^x$ is (0, 1).