1) THE SINE LAW

Solving for side

$$\frac{a}{sinA} = \frac{b}{sinB}$$

Solving for angle

$$\frac{\sin A}{a} = \frac{sinB}{b}$$

TYPE 1: ANGLE ANGLE SIDE

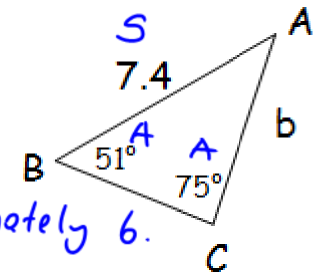
Solve for side b to the nearest one decimal place.

$$\sin 51 \cdot \frac{b}{\sin 51} = \frac{7.4}{\sin 75} \cdot \sin 51$$

$b = 5.953.....$

$b \doteq 6$

$\therefore$ b is approximately 6.



TYPE 2: ANGLE (NOT CONTAINED) SIDE SIDE

Solve for angle θ to the nearest degree.

$$5 \cdot \frac{\sin\theta}{5} = \frac{\sin 50}{8} \cdot 5$$

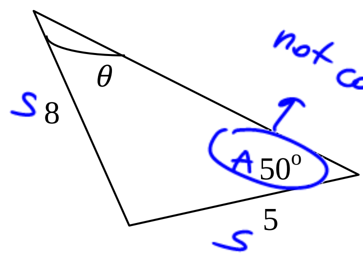
more accurate

$$\sin\theta = \frac{5\sin 50}{8}$$

$$\sin^{-1}\left(\frac{5 \cdot \sin 50}{8}\right) = \theta$$

$\theta \doteq 29$

$\therefore \theta$ is 29°.



contained S

2) THE COSINE LAW

$$\underbrace{c^2}_{\text{SIDE TO FIND}} = \underbrace{a^2 + b^2}_{\text{OTHER SIDES}} - 2ab\underbrace{cosC}_{\text{COS RATIO OF THE OPPOSITE ANGLE OF THE SIDE TO FIND}}$$

do at once

The cosine law is used to solve any triangle when given:

SIDE – CONTAINED ANGLE SIDE – (SAS)

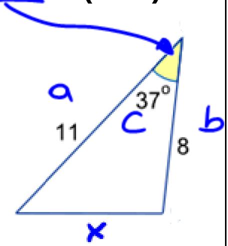
$c^2 = a^2 + b^2 - 2ab\cos C$

$x^2 = 11^2 + 8^2 - 2 \cdot 11 \cdot 8 \cdot \cos 37$

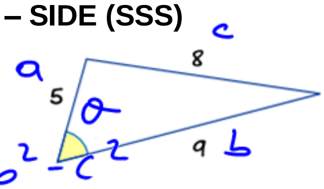
$\sqrt{x^2} = \sqrt{44.4402}$

$x \doteq 6.7$

$\therefore$ x is approximately 6.7 units.



SIDE – SIDE – SIDE (SSS)



$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos\theta = \frac{5^2 + 9^2 - 8^2}{2 \cdot 5 \cdot 9}$$

$$\cos\theta = \frac{42}{90}$$

leave as a fraction for more accuracy

$\cos^{-1}(42/90) = \theta$

$\theta \doteq 62$

$\therefore \theta$ is 62°.

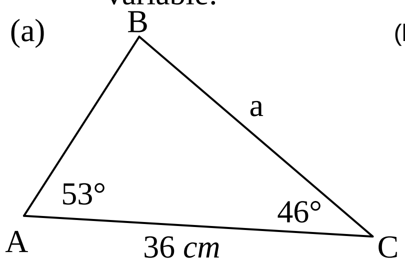
The Sine Law Practice

1. Solve for the given variable (correct to 1 decimal place) in each of the following:

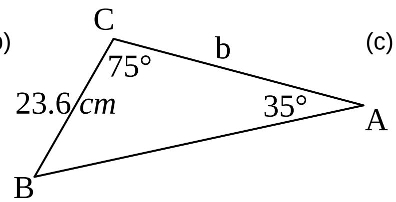
(a) $\frac{a}{\sin 35°} = \frac{10}{\sin 40°}$ (b) $\frac{65}{\sin 75°} = \frac{b}{\sin 48°}$ (c) $\frac{75}{\sin 55°} = \frac{c}{\sin 80°}$

2. For each of the following diagrams write the equation you would use to solve for the indicated variable:

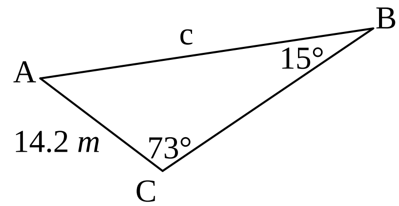
(a)



(b)



(c)



3. Solve for each of the required variables from Question #2.

4. For each of the following triangle descriptions you should make a sketch and then find the indicated side rounded correctly to one decimal place.

(a) In ΔABC, given that ∠A = 57°, ∠B = 73°, and AB = 24 cm. Find the length of AC

(b) In ΔABC, given that ∠B = 38°, ∠C = 56°, and BC = 63 cm. Find the length of AB

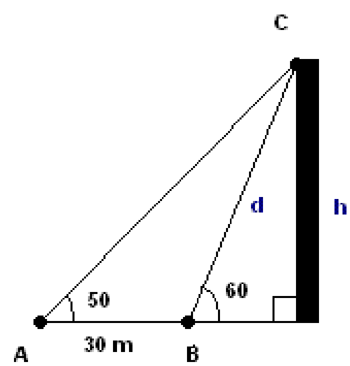
(c) In ΔABC, given that ∠A = 50°, ∠B = 50°, and AC = 27 m. Find the length of AB

(d) In ΔABC, given that ∠A = 23°, ∠C = 78°, and AB = 15 cm. Find the length of BC

(e) In ΔABC, given that ∠A = 55°, ∠B = 32°, and BC = 77 cm. Find the length of AC

(f) In ΔABC, given that ∠B = 14°, ∠C = 78°, and AC = 36 m. Find the length of BC

Challenge: The angle of elevation to the top C of a building from two points A and B on level ground are 50 degrees and 60 degrees respectively. The distance between points A and B is 30 meters. Points A, B and C are in the same vertical plane. Find the height h of the building (round your answer to the nearest unit).



$\frac{d}{\sin 50} = \frac{30}{\sin 10} \Rightarrow d = \frac{30 \sin 50}{\sin 10} = \boxed{132.34}$

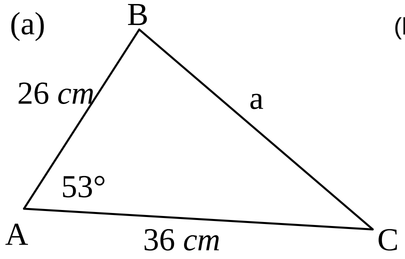
$\sin 60 = \frac{h}{d} \Rightarrow h = d.\sin 60$
$= 132.34 \sin 60$
$\boxed{h = 114.62}$

∴ The height is 115m.

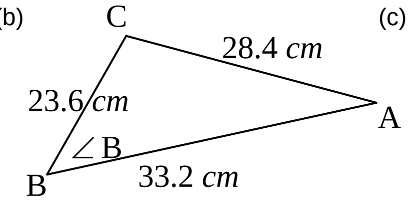
The Cosine Law Practice

1. For each of the following diagrams write the equation you would use to solve for the indicated variable:

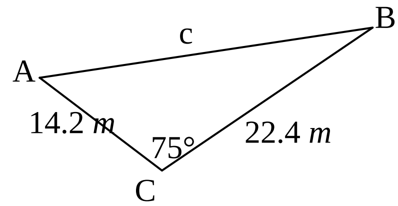
(a)



(b)



(c)



2. Solve for each of the required variables from Question #1.

3. For each of the following triangle descriptions you should make a sketch and then find the indicated value.

(a) In ΔABC, given that AB = 24 cm, AC = 34 cm, and ∠A = 67°. Find the length of BC

(b) In ΔABC, given that AB = 15 m, BC = 8 m, and ∠B = 24°. Find the length of AC

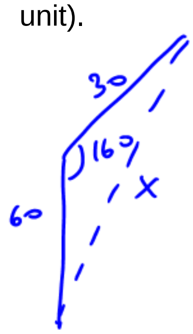
(c) In ΔABC, given that AC = 10 cm, BC = 9 cm, and ∠C = 48°. Find the length of AB

(d) In ΔABC, given that ∠A = 24°, AB = 18.6 m, and AC = 13.2 m. Find the length of BC

(e) In ΔABC, given that AB = 9 m, AC = 12 m, and BC = 15 m. Find the measure of ∠B.

(f) In ΔABC, given that AB = 18.4 m, BC = 9.6 m, and AC = 10.8 m. Find the measure of ∠A.

CHALLENGE: A ship leaves port at 1 pm traveling north at the speed of 30 miles/hour. At 3 pm, the ship adjusts its course 20 degrees eastward. How far is the ship from the port at 4pm? (round to the nearest unit).

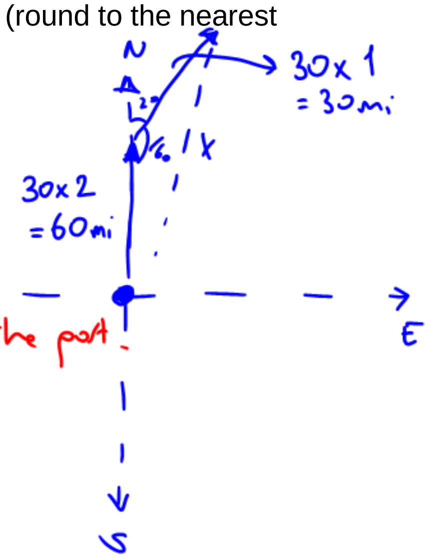


$x^2 = 30^2 + 60^2 - 2 \cdot 30 \cdot 60 \cdot \cos 160$

$\sqrt{x^2} = \sqrt{7882.8934}$

$x \doteq 89$

∴ The boat is app. 89 miles away from the port!



Solutions:

1. (a) $a^2 = (36)^2 + (26)^2 - 2(36)(26) \cdot \cos 53°$

   (b) $(28.4)^2 = (23.6)^2 + (33.2)^2 - 2(23.6)(33.2) \cdot \cos \angle B$

   (c) $c^2 = (22.4)^2 + (14.2)^2 - 2(22.4)(14.2) \cdot \cos 75°$

2. (a) 29.1 *cm* (b) 57° (c) 23.2 *m*
3. (a) 33.1 *cm* (b) 8.4 *m* (c) 7.8 cm (d) 8.5 *m* (e) 53° (f) 24°