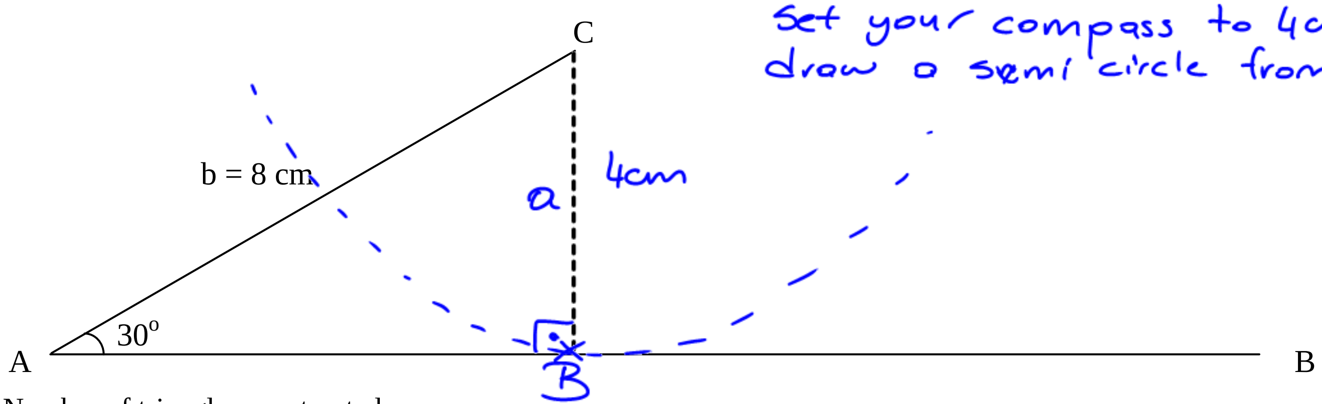
Ambiguous is defined as "having more than one possible meaning". In trigonometry, ambiguity exists for certain problems using the Sine Law.
HINT: Cut pieces of Cappellinis (thin spaghettis) for hands on experience or use a compass.

1) THE ACUTE CASE: $in \Delta ABC, \angle A < 90^\circ$,

CASE 1: When $\angle$ A = 30˚, b = 8 cm, how many triangle(s) can you draw with the missing side that equals to 4 cm? The length of c (side AB) is unspecified (meaning you can change its length to form the triangle).

Set your compass to 4cm and draw a semi circle from vertex C



Number of triangles constructed
1

Did you notice anything special about this line?
It is perpendicular to AB

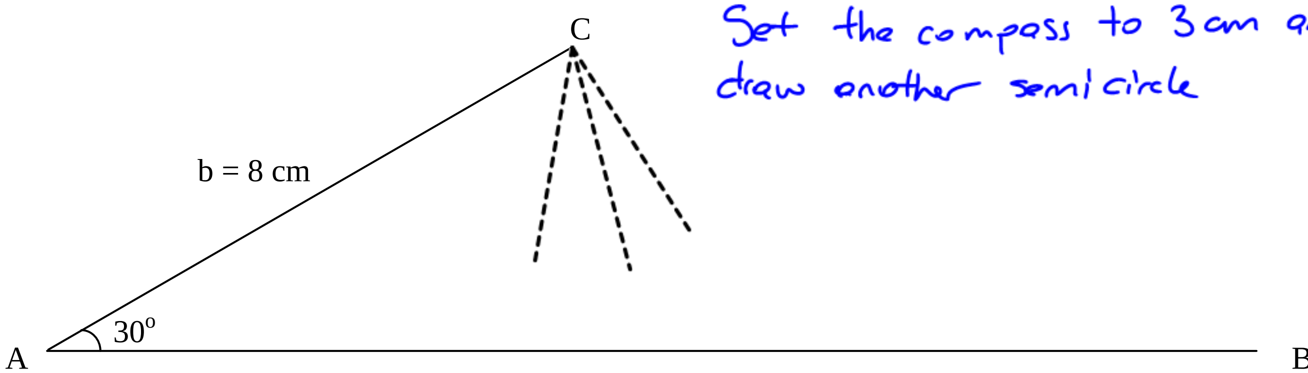
Write an algebraic expression for side a using b and sin A.
$\sin A = \frac{a}{b} \quad \therefore a = b \sin A$ → this calculates you the height

a is the height

CONCLUSION:
If $\angle$ A is acute (between 0 and 90) and $a = b\sin A$, there will be only ONE triangle which is a right triangle.
Therefore, no ambiguity exists.

CASE 2: How many triangle(s) can you draw if we set the missing side to 3 cm?
The length of c (side AB) is unspecified (meaning you can change its length).

Set the compass to 3cm and draw another semi circle

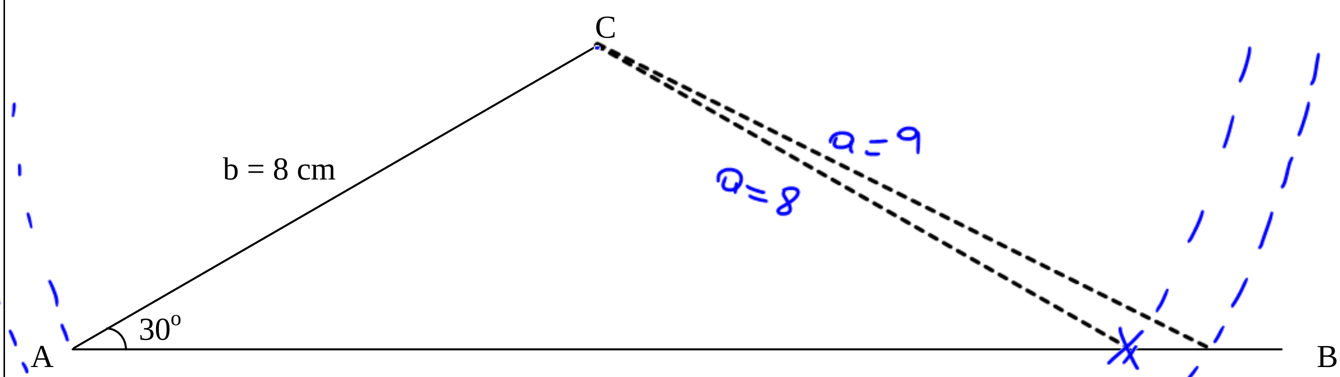


Number of triangles constructed?
none

Did you notice anything special about this line?
The line is shorter than height (height is the shortest distance)

CONCLUSION:
If $\angle$ A is acute and "a" is less than the height (a < b sin A), there will be NO triangles. In other words, you cannot draw a triangle with a side that is shorter than the shortest side.

CASE 3: How many triangle(s) can you draw if we set the missing side to 8 cm or 9 cm?
The length of c (side AB) is unspecified (meaning you can change its length).

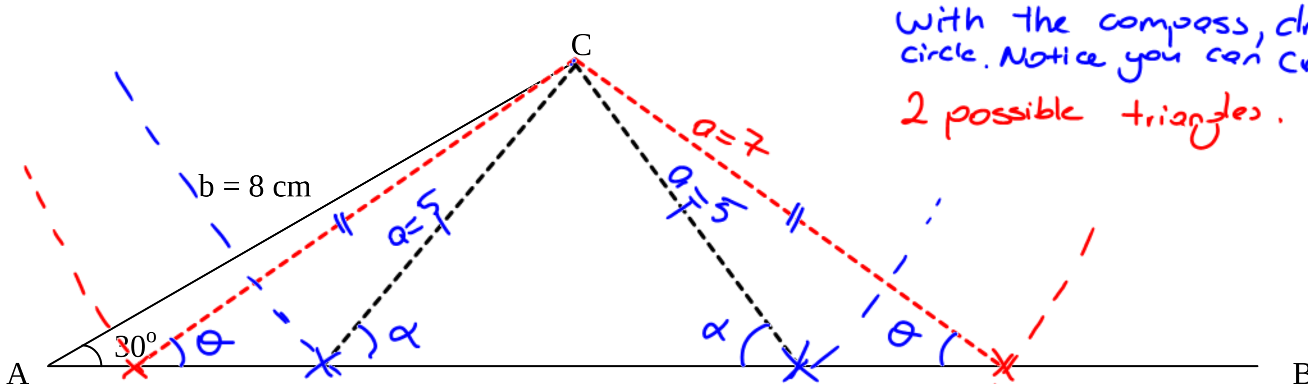


Number of triangles constructed when **a ≥ b** (a is greater than or equal to b)?
ONE

If ∠A is acute and **a ≥ b**, there will be only ONE triangle that can be constructed. **Therefore, no ambiguity exists.**

CASE 4: How many triangle(s) can you draw if we set the missing side to 5 cm then 7 cm?
The length of c (side AB is unspecified (meaning you can change its length).

With the compass, draw a semi circle. Notice you can construct 2 Δs.
2 possible triangles.

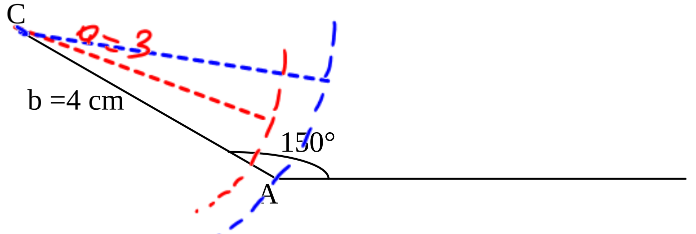


Number of triangles constructed when a is between the height and b ($b\sin A < a < b$)?
2

If ∠ A is acute and **b sin A < a < b**, there will be two triangles that can be constructed.
Therefore, an AMBIGUOUS Case exists.

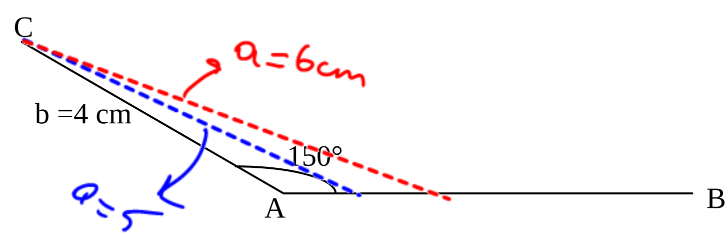
2) THE OBTUSE CASE: $in\Delta ABC, \angle A \geq 90^\circ$,

Case 5:
How many triangles can you draw when the missing side is 3 cm then 4cm? The length of c (side AB) is unspecified.



If ∠ A is obtuse and the missing side is less than side AC (a ≤ b), there will be no triangles.

Case 6:
How many triangles can you draw when the missing side is 5cm then 6 cm? The length of c (side AB) is unspecified.



If ∠ A is obtuse and the missing side is greater than side AC (**a > b**), there will be only one triangle.
Note that there is no ambiguous case if ∠ A ≥ 90°

NOTE: We always question if the side opposite to given angle creates a case.

SUMMARY

Đ A < 90°	a < b sin A	no triangle	THE ACUTE CASE
	a = b sin A	one triangle (right angle)	
	b sin A < a < b	ambiguous case (two triangles)	
	a ≥ b :	one triangle	
Đ A ≥ 90°	a ≤ b	no triangle	THE OBTUSE CASE
	a > b	one triangle	

When you are given two sides and an angle not in between those sides, you need to be on the lookout for the ambiguous case.

How to determine if there is a 2nd valid angle:
1. See if you are given two sides and the angle not in between (SSA). This is the situation that may have 2 possible answers.
2. Find the value of the unknown angle.
3. Once you find the value of your angle, subtract it from 180° to find the possible second angle.
4. Add the new angle to the original angle. If their sum is less than 180°, you have two valid answers. If the sum is over 180°, then the second angle is not valid.

1) In ΔQRS, ∠Q=105º, r=15, q=20. Determine the number of triangles possible. Solve the triangle(s) if possible.

Since this's an obtuse case, side q needs to be greater than r.
There's one triangle.

$15 \cdot \frac{\sin\theta}{15} = \frac{\sin 105}{20} \cdot 15$

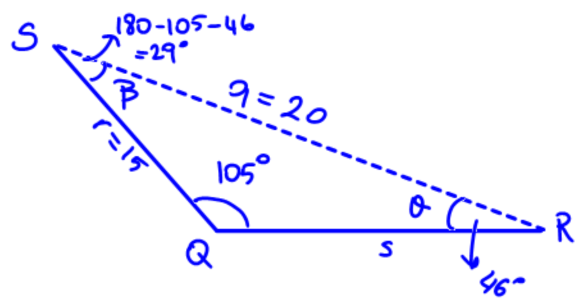
$\sin\theta \doteq 0.7244$

$\sin^{-1}(0.7244) = \theta$

$\boxed{\theta \doteq 46°}$

$\sin 29 \cdot \frac{s}{\sin 29} = \frac{20}{\sin 105} \cdot \sin 29$

$\boxed{s \doteq 10}$

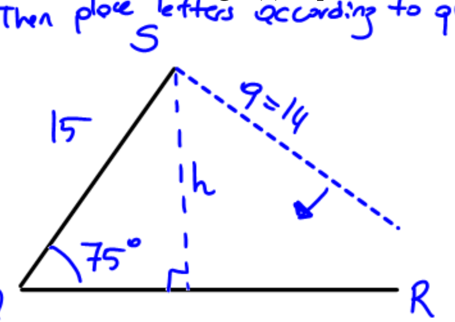


∴ Side s is 10
θ = 46
β = 29

2) In ΔQRS, ∠Q=75º, r=15, q=14. Determine the number of triangles possible. Solve the triangle(s) if possible.

1) Acute, possible ambiguous case. Draw "∠" first. Then place letters according to question
2) Then, let's check if the side q is long enough to be the height. If not, a triangle cannot be formed.

height = 15 · sin 75
h ≐ 14.5



∴ Since side q is shorter than the shortest side, a triangle cannot be formed

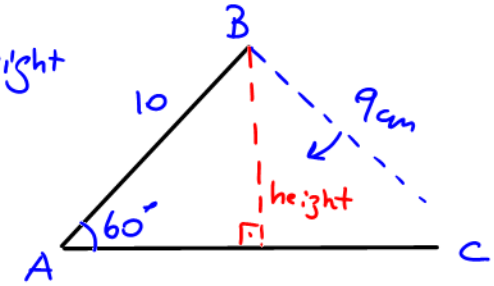
3) In ΔABC, $\angle A = 60^{\circ}$, c = 10cm and a = 9cm. Determine the number of triangles possible. Solve the triangle(s) if possible.

1) $\angle A$ is acute, possible ambiguous case.

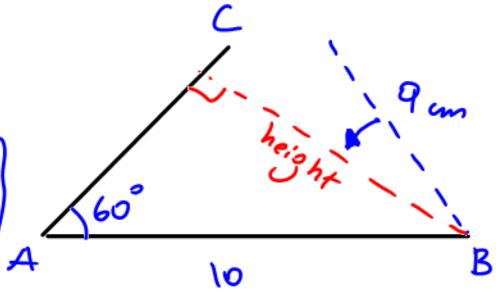
2) Let's check if side a is long enough to be a height

$h = 10 \cdot \sin 60$

$\boxed{h \doteq 8.7\text{ cm}}$



3) side in question is greater than height but less than side c; therefore it's an ambiguous case.



either triangle is correct. It just depends on how you look at the triangle. We're trying to find out if the side opposite to the given angle will create an ambiguous case.

Put this note b/c you may draw the triangle like this one

AMBIGUOUS CASE

ACUTE TRIANGLE

Triangle 1

$10 \cdot \frac{\sin\theta}{10} = \frac{\sin 60}{9} \cdot 10$

$\sin\theta \doteq 0.9623$

$\sin^{-1}(0.9623) = \theta$

$\boxed{\theta \doteq 74}$

$\alpha = 180 - 60 - 74$

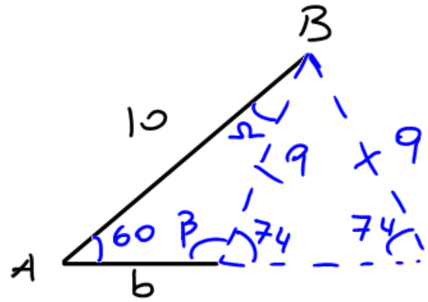
$\boxed{\alpha \doteq 46^{\circ}}$

$\sin 46 \cdot \frac{b}{\sin 46} = \frac{9}{\sin 60} \cdot \sin 46$

$b \doteq 7.5$

$\therefore$ b is 7.5cm.
θ is 74°
α is 46°

OBTUSE TRIANGLE



$\beta = 180 - 74$

$\boxed{\beta = 106^{\circ}}$

$\Omega = 180 - 60 - 106$

$\boxed{\Omega \doteq 14^{\circ}}$

$\sin 14 \cdot \frac{b}{\sin 14} = \frac{10}{\sin 106} \cdot \sin 14$

$b \doteq 2.5$

$\therefore$ b is 2.5 cm
β is 106°
Ω is 14°.

COMPLETE: p.318 #2-5, 11, 13, 14