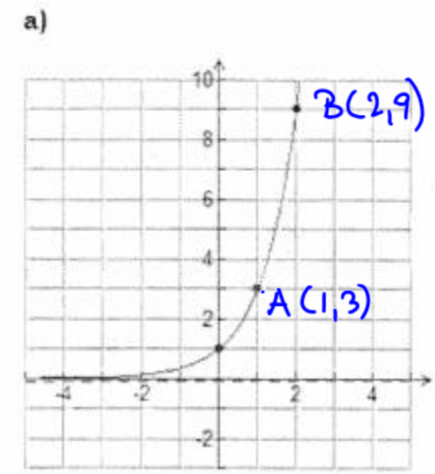
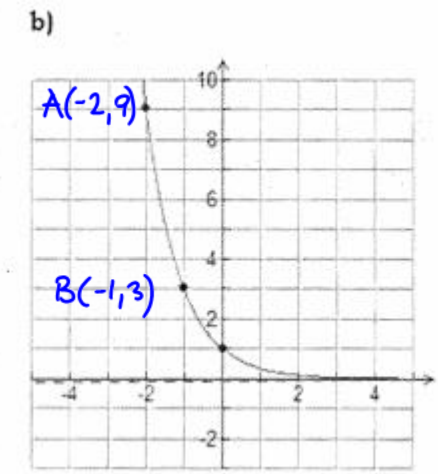
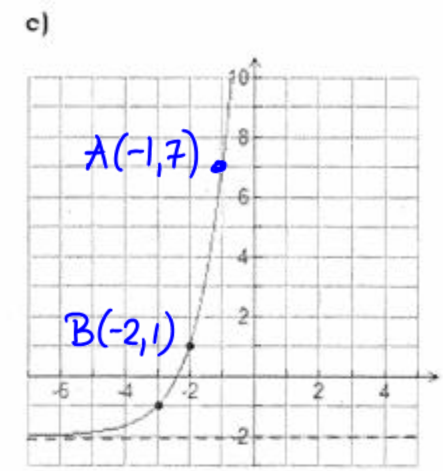
Determining the Equation of an Exponential Function

1. Choose 2 points from the transformed graph
2. Determine c from the y-value of the horizontal asymptote
3. Subtract c from the y-values of the chosen points (to translate back to the x-axis)
4. Substitute the (x,y) pairs into $y = ab^x$ to create two equations.
5. Rearrange and solve the system using substitution.
6. State the answer in the form $y = \pm ab^x + c$.

Ex1. Determine an equation for each of the following graphs in the form of $y = ab^x + c$







a)

$c=0$

① Using A(1,3)

$y=ab^x+c$

$3=ab$

② $y=ab^x$ B(2,9)

$9=ab^2$

③ Divide ② by ①

$\frac{9}{3}=\frac{ab^2}{ab}$

$\boxed{3=b}$

④ sub "3" for "b"

$3=a(3)$

$\boxed{1=a}$

$\therefore \underline{y=3^x}$

b)

$c=0$

① A(-2,9)

$9=ab^{-2}$

② B(-1,3)

$3=ab^{-1}$

③ divide ② by ①

$\frac{3}{9}=\frac{\cancel{a}b^{-1}}{\cancel{a}b^{-2}}$

$\boxed{\frac{1}{3}=b}$

④ sub "$\frac{1}{3}$" for "b"

$3=a\left(\frac{1}{3}\right)^{-1}$

$3=a(3)$

$\boxed{a=1}$

$\therefore y=\left(\frac{1}{3}\right)^x$

c)

$c=-2$

① $y=ab^x-2$ A(-1,7)

$7=ab^{-1}-2$

$\boxed{9=ab^{-1}}$

② $y=ab^x-2$ B(-2,1)

$1=ab^{-2}-2$

$\boxed{3=ab^{-2}}$

③ divide

$\frac{3}{9}=\frac{ab^{-2}}{ab^{-1}}$

$\frac{1}{3}=b^{-1}$

$\boxed{3=b}$

$\therefore 9=a\left(\frac{1}{3}\right)$

$\boxed{a=27}$

$\therefore y=27(3)^x-2$

Ex2. Find the exponential function through (2, 16) and (6, 256) that has a horizontal asymptote at y = 0.

$y = ab^x + 0$

① $16 = ab^2$ A(2, 16)

② $256 = ab^6$ B(6, 256)

③ $\frac{256}{16} = \frac{ab^6}{ab^2}$

$\sqrt[4]{16} = \sqrt[4]{b^4}$

$\boxed{2 = b}$

④ $16 = a(2)^2$

$16 = 4a$

$\boxed{a = 4}$

$\therefore y = 4(2)^x$

Ex3. Find the exponential function through (2, 10) and (4, 22) that has a horizontal asymptote at y = 4.

$y = ab^x + 4$

① $10 = ab^2 + 4$

$\boxed{6 = ab^2}$

② $22 = ab^4 + 4$

$\boxed{18 = ab^4}$

③ $\frac{18}{6} = \frac{ab^4}{ab^2}$

$\sqrt{3} = \sqrt{b^2}$

$\sqrt{3} = b$

④ $6 = a(\sqrt{3})^2$

$6 = 3a$

$\boxed{a = 2}$

$\therefore y = 2(\sqrt{3})^x + 4$

$= 2(3^{\frac{1}{2}})^x + 4$

$= 2(3)^{\frac{1}{2}x} + 4$

Ex4. Find an exponential function that passes through (3, 12.5) and (4, 11.25) and has a horizontal asymptote of y = 10.

Step1: $y = ab^x + 10$

$12.5 = ab^3 + 10$

$2.5 = ab^3$

Step2 : $11.25 = ab^4 + 10$

$1.25 = ab^4$

Step3 $\frac{1.25}{2.5} = \frac{ab^4}{ab^3}$

$\boxed{0.5 = b}$

Step4 : $2.5 = a(0.5)^3$

$2.5 = a(0.125)$

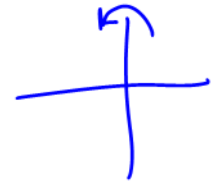
$\boxed{20 = a}$

$\therefore y = 20(0.5)^x + 10$

$\downarrow$

$y = 20(\frac{1}{2})^x + 10$

Ex5. The graph of $f(x) = 2^x$ is compressed vertically by a factor of $\frac{1}{2}$, reflected in the y-axis, and translated right 4 units and downward 5 units.



a) Write the equation of the new function.

$a = \frac{1}{2}$ $k = -1$ $d = 4$ $c = 5$

$$y = \frac{1}{2}\left[2^{-(x-4)}\right] - 5$$

b) State the domain, range, y-intercept and equation of the horizontal asymptote.

→ $D = \{x \in R\}$

→ $R = \{y \in R \mid y > -5\}$

→ $y = -5$

→ sub '0' for x → $y = \frac{1}{2}\left(2^{-(0-4)}\right) - 5$

$= 0.5(2^4) - 5$

$= 8 - 5$

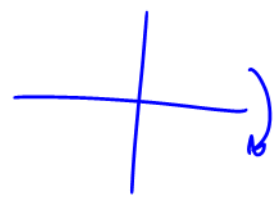
$= 3$

$\therefore (0, 3)$

Ex6. The equation of the function that represents $f(x) = (\frac{1}{4})^x$ after it is compressed horizontally by a factor of $\frac{1}{2}$, reflected in the x-axis, and shifted 4 to the left and 6 units up.

a) Write the equation of the new function.

$$f(x) = -\left[\frac{1}{4}^{2(x+4)}\right] + 6$$



b) State the domain, range, and equation of the horizontal asymptote.

→ $D = \{x \in R\}$

$R = \{y \in R \mid y < 6\}$

$y = 6$

sub "0" for "x"

$y = -\left[\frac{1}{4}^{2(0+4)}\right] + 6$

$= -\left(\frac{1}{4}\right)^8 + 6$

$\doteq 5.9985$

$\therefore (0, 5.9985)$