8.2 & 8.3: Compound Interest: Present and Future Value

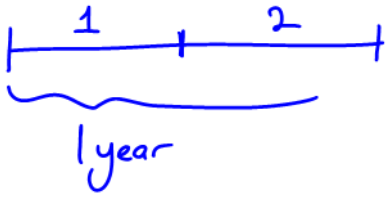
Chapter 8: Financial Mathematics

MCR3U1

Compound Interest

→ $ invested / borrowed

- Compound interest is interest which is added to the original principal.
- You are getting interest on top of interest when you invest into an account that offers compound interest.
- The investment can be compounded by the following periods:
  - Semi-annually – 2
  - Quarterly – 4
  - Bi-weekly- 26
  - Semi-monthly- 24
  - Annually – 1
  - Weekly- 52



Developing the Compound Interest Formula

Example 1: You invest $2000 in an account that earns 7.5% interest per year compounded annually for 5 years.

P

a) Fill out the table below:

Year	Principle	Interest	Total amount
0	2000	0	2000
1	2000	2000 x 0.075 = 150	2000 + 150 = 2150
2	2150	161.25	2150 + 161.25 = 2311.25
3			
4			
5			2871.25

b) What is the investment worth after 5 years?

It's worth $2871.25

2, 4, 8

$r = \frac{4}{2}$ or $\frac{8}{4}$

c) Calculate the investment using simple interest. How much more do you make if the money is compounded?

$I = Prt$

$= 2000(0.075)(5)$

$= 750$

∴ You make $121.25

Interest Earned from Comp $= 2871.25 - 2000$

$= \$871.25$

d) Find the common ratio of the total amount. Create an exponential formula that represents the compound interest above.

2000, 2150, 2311.25 .... 2871.25

b is common ratio $= \frac{2150}{2000}$

$= 1.075$

$y = ab^x$

$y = 2000(1.075)^x$

$1 + r$

$1 + 0.075$ → 7.5%

Compound Interest Formula

You can calculate compound interest by using the formula

$A = P(1 + i)^n$ where..

Type = - - - - - -

A = Total amount of investment (or future value)→FV $= PV(1+i)^n$

P = Principal (or present value)→PV $= FV(1+i)^{-n}$

i = Interest rate as a decimal, per compounding period

- **i = r ÷ N** where **r** is the interest rate per annum
  N is the number of compounding periods per annum

n = Total number of compounding periods

- **n = yN** where **y** – number of years

$\frac{FV}{(1+i)^n} = \frac{PV(1+i)^n}{(1+i)^n}$

$\frac{FV}{(1+i)^n} = PV$

$PV = \frac{FV}{(1+i)^n}$ OR $PV = FV(1+i)^{-n}$

$\frac{1}{2}$ is 2^{-1}

Example 2: Calculate the interest rate (i not R) for an 8%/a investment compounded:

a) semi-annually b) weekly c) monthly

4% 4% 8%

$i = \frac{0.08}{2} = 0.04$

$i = \frac{0.08}{52}$

$\frac{0.08}{12}$

Example 3: Calculate the number of compounding periods in the following investments:

a) Compounded quarterly for 5 years.

1 year = 4 comp. period

$4 \times 5 = 20$

b) Compounded semi-annually for 18 months

1 2 1 year

3 and 6 months

3

c) Compounded bi-weekly for 2 years

$26 \times 2 = 52$

Example 4: Calculate the amount of an investment if $500 is invested at 3% interest compounded quarterly for 3 years.

Type : comp quarterly (4)

A : ?

P : 500

i : 3%/a = 0.03/a = 0.03 ÷ 4

n : 3 years X 4 = 12

$A = P(1+i)^n$

$= 500(1 + 0.03 \div 4)^{12}$

$= 546.90$

∴ The amount will be $546.90

Example 5: What is the amount of an investment if $650 is invested at 2.45% interest compounded monthly for 3 years?

Type = compounded monthly

A = ?

P = 650

i = 0.0245/a ÷ 12

n = 3 years X 12 = 36

$FV = PV(1+i)^n$

$= 650(1 + 0.0245 \div 12)^{36}$

$= 699.52$

∴ The amount will be $699.52

Example 6:

Natalie invests $18 000 at 8%/a, compounded semiannually.

(a) Determine the value of the investment after four years.

(b) Find the interest at this time.

T = Semi annual (2)

A = ?

P = 18 000

i = 0.08/2

n = 2 × 4

a) $A = P(1+i)^n$

$= 18000(1+0.08/2)^8$

$= 24\,634.24$

∴ amount is $24 634.24

b) $24\,634.24 - 18\,000$

$= 6634.24$

∴ interest is 6634.24

Example 7:

Determine the present value of an investment that will be worth $5000 in ten years. The interest rate is 4.8%/a, compounded quarterly.

Type = comp quarterly (4)

A = 5000

P = ?

i = 0.048 ÷ 4

n = 10 × 4 = 40

$P = A(1+i)^{-n}$

$= 6000(1+0.048 \div 4)^{-40}$

$= 3102.77$

∴ The present value is $3102.77

by: Alessia Spano

Pg. 490 #4ace, 5, 6, 9-11 (compound interest future value)

Pg. 498 #3ac, 5, 6, 8, 9 (compound interest present value)