Trigonometric Ratios for Special Angles

There are 2 kinds of right triangles that create interesting trigonometric ratios. They are:

The $45° - 45° - 90°$ Triangle

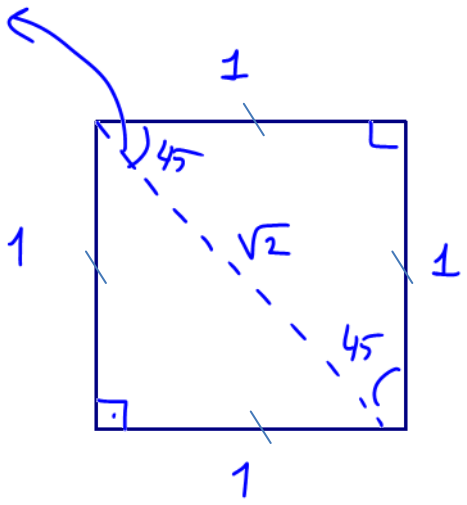
rationalize

$$\sin 45° = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\cos 45° = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\tan 45° = \frac{1}{1} = 1$$

$x^2 = 1^2 + 1^2$
$x^2 = 2$
$x = \sqrt{2}$

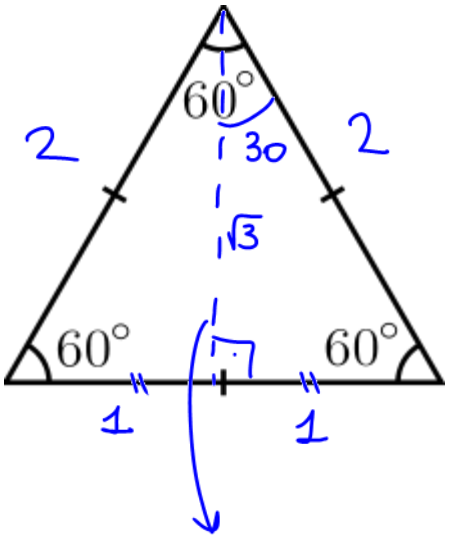


The $30° - 60° - 90°$ Triangle (version 1)

$$\sin 30° = \frac{1}{2}$$

$$\cos 30° = \frac{\sqrt{3}}{2}$$

$$\tan 30° = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$



The $30° - 60° - 90°$ Triangle (version 2)

$$\sin 60° = \frac{\sqrt{3}}{2}$$

$$\cos 60° = \frac{1}{2}$$

$$\tan 60° = \sqrt{3}$$

$x^2 = 2^2 - 1^2$
$x^2 = 3$
$x = \sqrt{3}$

Trigonometric Ratios for Special Angles continued...

- SPECIAL ANGLES and their values can be used to solve a variety of problems. Suggestion: Don't worry about memorizing the trig ratios, you can always MATHEMATISE them if you know the triangles.

NOTE: $(\sin x)^2 = \sin^2 x$ **NOT** $\sin x^2$

EXAMPLE 1 – Evaluate the following expressions using exact values – ***no calculator & no decimals!***

a) $\sin 30° + \cos 30°$

$= \frac{1}{2} + \frac{\sqrt{3}}{2}$

$= \frac{1+\sqrt{3}}{2}$

b) $\tan 60° - \cos 45°$

$= \sqrt{3} - \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{2\sqrt{3}}{2 \cdot 1} - \frac{\sqrt{2}}{2}$

$= \frac{2\sqrt{3} - \sqrt{2}}{2}$

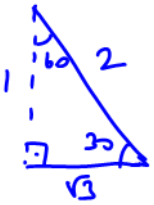
c) $\sin 60° + \tan 30°$

$= \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{3\sqrt{3}}{3 \cdot 2} + \frac{2\sqrt{3}}{2 \cdot 3}$

$= \frac{3\sqrt{3} + 2\sqrt{3}}{6}$

$= \frac{5\sqrt{3}}{6}$



d) $\tan^2 60° + 2\tan^2 45°$

$= (\tan 60)^2 + 2(\tan 45)^2$

$= (\sqrt{3})^2 + 2(1)^2$

$= 3 + 2(1)$

$= 5$

EXAMPLE 2 – Using exact values, determine θ if $0° \le \theta \le 90°$

a) $\sqrt{2}\sin\theta - 1 = 0$

$\frac{\sqrt{2}\sin\theta}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\sin\theta = \frac{1}{\sqrt{2}}$

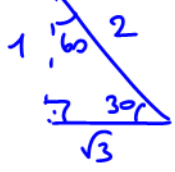
$\theta = 45°$

b) $2\cos\theta - \sqrt{3} = 0$

$\frac{2\cos\theta}{2} = \frac{\sqrt{3}}{2}$

$\cos\theta = \frac{\sqrt{3}}{2}$

$\theta = 30°$



c) $4\cos^2\theta - 1 = 0$

$4\cos^2\theta = 1$

$\sqrt{\cos^2\theta} = \sqrt{\frac{1}{4}}$

$\cos\theta = \pm 1/2$

We'll work with $1/2$ b/c θ is in the 1st quad

$\cos\theta = 1/2$

$\theta = 60°$

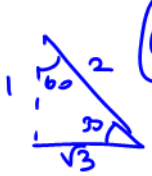
d) $\tan^2\theta - 3 = 0$

$\sqrt{\tan^2\theta} = \sqrt{3}$

$\tan\theta = \pm\sqrt{3}$

$\tan\theta = \sqrt{3}$

$\theta = 60°$



EXTENDING

THREE RECIPROCAL TRIG RATIOS

$$\csc\theta = \frac{1}{\sin\theta}$$

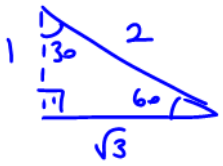
$$\sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

a. If $\cot\propto = \sqrt{3}$, calculate $(\sin\propto)(\cos\propto) - \cos^2\propto$ exactly.

$\frac{1}{\tan\alpha} = \sqrt{3} \Rightarrow \tan\alpha = \frac{1}{\sqrt{3}}$

$\alpha = 60$



$= (\sin 60)(\cos 60) - (\cos 60)^2$

$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)^2$

$= \frac{\sqrt{3}}{4} - \frac{3}{4}$

$= \frac{\sqrt{3}-3}{4}$ //

b. If $\csc\beta = 2$, calculate $\frac{\tan\beta}{\sec\beta} - \sin^2\beta$ exactly.