

Unit 1 Test VERSION A
Functions and Transformations

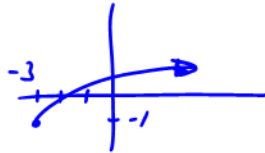
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PART 1 - KNOWLEDGE: [18 MARKS]

1. Determine whether each of the following are functions and then state the domain and range for the relations:

a) $y = \sqrt{3(x+3)} - 1$ [3 marks]

$x = 0$
 $y = 2$

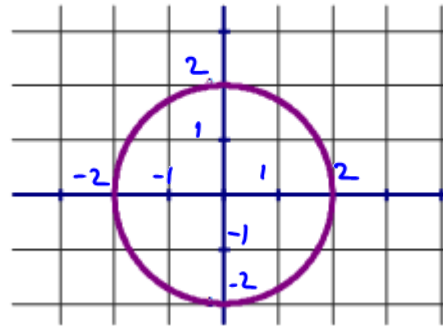


Is this a function? YES

D = $\{x \in \mathbb{R} \mid x > -3\}$

R = $\{y \in \mathbb{R} \mid y > -1\}$

b) [3 marks]



Is this a function? NO

D = $\{x \in \mathbb{R} \mid -2 \leq x \leq 2\}$

R = $\{y \in \mathbb{R} \mid -2 \leq y \leq 2\}$

2. Given $f(x) = \frac{1}{x+4} - 2$ find: [3 marks]

a) $f(x^2) = \frac{1}{x^2+4} - 2$

b) $f(3) = \frac{1}{3+4} - 2$
 $= \frac{1}{7} - 2$
 $= \frac{1-14}{7} = -\frac{13}{7}$

3. Consider the relation $f(x) = 2x^2 - 4$. Find $f^{-1}(x)$. Be sure to use proper notation. [3 marks]

$y = 2x^2 - 4$

$x = 2y^2 - 4$

$\frac{x+4}{2} = \frac{2y^2}{2}$

$\sqrt{\frac{x+4}{2}} = y$

$\sqrt{\frac{x}{2} + 2} = y$

$f^{-1}(x) = \sqrt{\frac{x}{2} + 2}$ or $f^{-1}(x) = -\sqrt{\frac{x}{2} + 2}$

4. Graph the function $f(x) = 2(x + 6)^2 + 2$ and its inverse. [6 marks]

$$(x, y) \rightarrow (x - 6, 2y + 2)$$

$$A(-2, 4) \rightarrow A'(-8, 10)$$

$$B(-1, 1) \rightarrow B'(-7, 4)$$

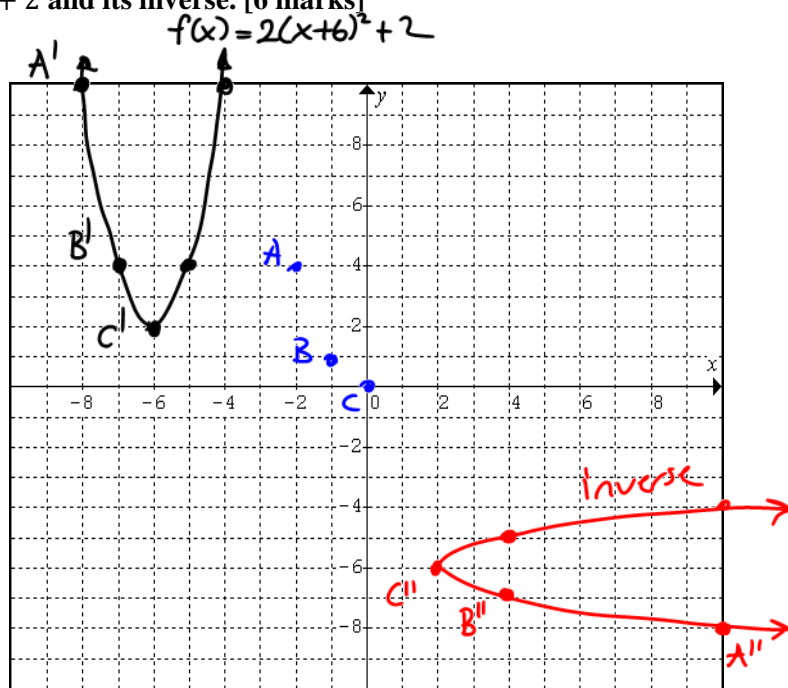
$$C(0, 0) \rightarrow C'(-6, 2)$$

INVERSE

$$A''(10, -8)$$

$$B''(4, -7)$$

$$C''(2, -6)$$



PART 2 - APPLICATION: [18 MARKS]

1. Select the appropriate description of how the graph of each function is derived from the graph of $y = x^2$. Place the number next to the correct answer. [6 marks]

a) $y = \left[\frac{1}{3}(x)\right]^2$ 8

b) $y = (3x)^2$ 9

c) $y = x^2 - 3$ 6

d) $y = (x - 3)^2$ 3

e) $y = -(x + 3)^2$ 7

f) $y = -x^2 + 3$ 11

- (1) vertical compression by a factor of $\frac{1}{3}$
- (2) vertical expansion by a factor of 3
- (3) horizontal translation 3 units right
- (4) horizontal translation 3 units left
- (5) vertical translation 3 units up
- (6) vertical translation 3 units down
- (7) reflection in x-axis and horizontal translation 3 units left
- (8) horizontal expansion by a factor of 3
- (9) horizontal compression by a factor of $\frac{1}{3}$
- (10) horizontal translation 3 units right and vertical translation 3 units up
- (11) reflection in x-axis and vertical translation 3 units up

2. Given the function $f(x) = \sqrt{x}$, sketch the parent function and $y = -2\sqrt{(x+3)} + 4$ without a table of values. [5 marks]

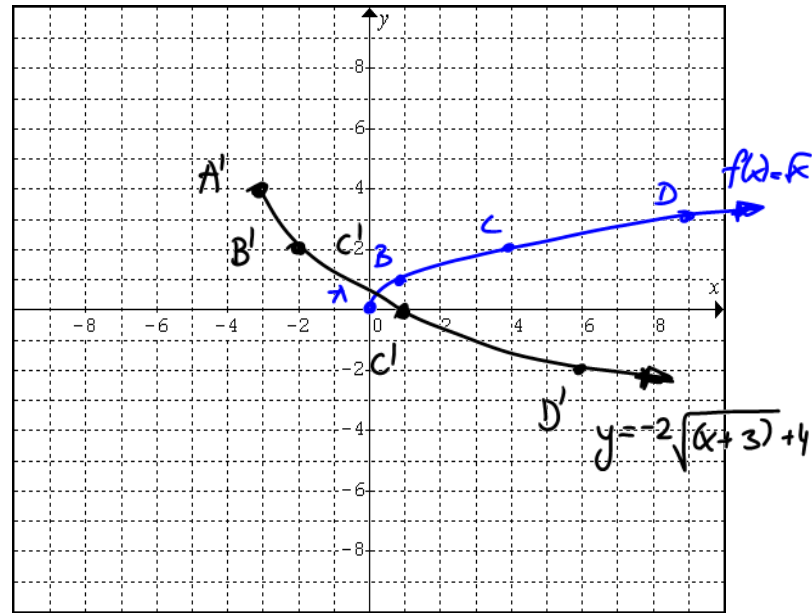
$$(x, y) \rightarrow (x-3, -2y+4)$$

$$A(0, 0) \rightarrow A'(-3, 4)$$

$$B(1, 1) \rightarrow B'(-2, 2)$$

$$C(4, 2) \rightarrow C'(1, 0)$$

$$D(9, 3) \rightarrow D'(6, -2)$$



3. Given $f(x)$ shown below, graph $g(x) = -2f(x-3) + 1$. [4 marks]

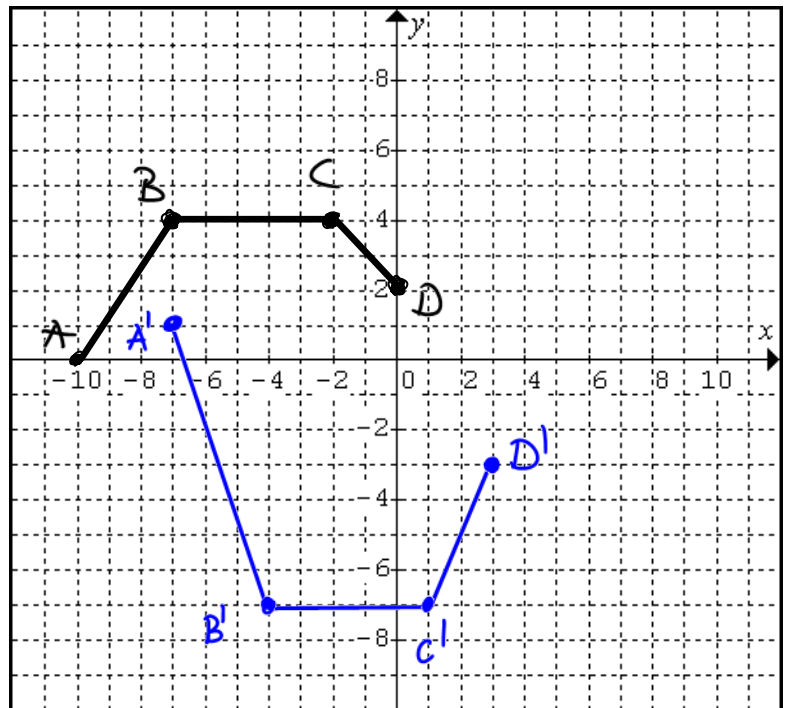
$$(x, y) \rightarrow (x+3, -2y+1)$$

$$A(-10, 0) \rightarrow A'(-7, 1)$$

$$B(-7, 4) \rightarrow B'(-4, -7)$$

$$C(-2, 4) \rightarrow C'(1, -7)$$

$$D(0, 2) \rightarrow D'(3, -3)$$



4. Given $f(x) = -2x^2 + 3x - 5$ and $g(x) = 7x - 9$ find $f(-4) - g(-2)$. [4 marks]

$$\begin{aligned} f(-4) &= -2(-4)^2 + 3(-4) - 5 \\ &= -2(16) - 12 - 5 \\ &= -32 - 12 - 5 \\ &= -49 \end{aligned}$$

$$\begin{aligned} g(-2) &= 7(-2) - 9 \\ &= -14 - 9 \\ &= -23 \end{aligned}$$

$$\begin{aligned} f(-4) - g(-2) &= -49 - (-23) \\ &= -49 + 23 \\ &= -26 \end{aligned}$$

PART 3 - THINKING: [8 MARKS]

Answer ONLY 2 out of 3 questions. Put an X through the questions you don't want marked.

1. If $f^{-1}(x) = -\sqrt{2x+4}$, find the equation of $f(x)$ and its domain and range. [4 marks]

$$\begin{aligned} y &= -\sqrt{2x+4} \\ (x)^2 &= (-\sqrt{2y+4})^2 \\ x^2 &= 2y+4 \end{aligned}$$

$$\begin{aligned} \frac{x^2}{2} - \frac{4}{2} &= \frac{2y}{2} \\ \frac{1}{2}x^2 - 2 &= y \end{aligned}$$

$$\begin{aligned} \checkmark D &= \{x \in \mathbb{R}\} \\ \checkmark R &= \{y \in \mathbb{R} \mid y \geq 2\} \end{aligned}$$

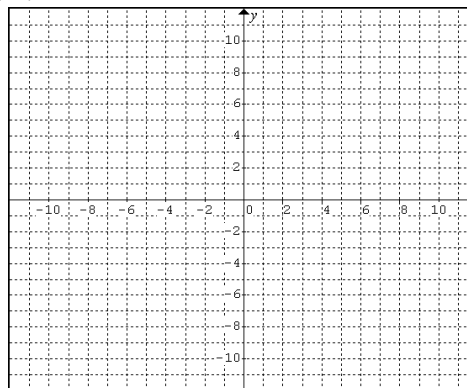
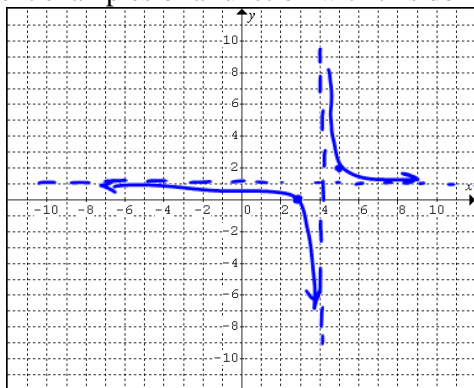


2. Identify a function that would have this domain and range. [4 marks]

$$\begin{aligned} D: \{x \in \mathbb{R}, x \neq 4\} \\ R: \{y \in \mathbb{R}, y \neq 1\} \end{aligned}$$

$$f(x) = \frac{1}{x-4} + 1$$

Draw two different examples of a function with this domain.



3. If $g(x) = 3x - 7$ and $h(x) = x^2$, find $h(g(2))$. [4 marks]

Step 1 $g(2) = 3(2) - 7$
 $= 1$

Step 2 $h(1) = 1^2$
 $= 1$

PART 4: COMMUNICATION (10 MARKS)

1. Express the following expression in words: $D: \{x \in \mathbb{R} | x \geq -4\}$ [2 marks]

x is an element of Real numbers such that x is greater than or equal to -4 .

2. Is every relation a function? Explain why or why not... [2 marks]

No it's not. In a relation an "x" value can repeat itself; however an "x" can only have one "y" value.

3. State the transformations, **in order**, that must be applied to the graph of $y = f(x)$ to obtain the following equation.

$$y = -4f\left(-\frac{1}{3}x\right) + 1 \quad [4 \text{ marks}]$$

R = Vertical reflection about "x" axis ✓
Horizontal reflection about "y" axis

S = Vertical stretch by a factor of 4 ✓
Horizontal stretch by a factor of 3 ✓

T = Translation 1 unit up ✓

4. Form Mark (you will be assigned a mark out of 4 for your mathematical form used throughout the quiz). [2]

	Level Four	Level Three	Level Two	Level One
Expression and organization of ideas and mathematical thinking	Expresses and organizes mathematical thinking with a high degree of effectiveness	Expresses and organizes mathematical thinking with considerable effectiveness	Expresses and organizes mathematical thinking with some effectiveness	Expresses and organizes mathematical thinking with limited effectiveness
Use of conventions, vocabulary, and terminology	Uses conventions, vocabulary and terminology with a high degree of effectiveness	Uses conventions, vocabulary and terminology with considerable effectiveness	Uses conventions, vocabulary and terminology with some effectiveness	Uses conventions, vocabulary and terminology with limited effectiveness