

Name: Solutions

MAP4C1

Date: _____

Trigonometry Review

Communication in all questions must include:

- Enough steps shown to clearly demonstrate thinking
- Solutions that are neat and easy to follow
- Proper use of mathematical symbols
- Equal signs aligned
- Units used as required
- Concluding statements for all word problems
- Fractions reduced to lowest terms
- Correct rounding.

You will be given the following information:

$$a^2 + b^2 = c^2$$

SOH CAH TOA

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

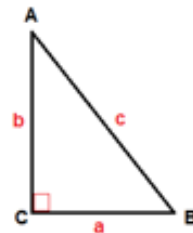
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

1. Write the three trigonometric ratios for A in the following triangle:

$$\sin A = \frac{a}{c}$$

$$\cos A = \frac{b}{c}$$

$$\tan A = \frac{a}{b}$$



2. For each triangle below:

- Explain which strategy/equation you would use to find the missing side (x).
- Find the missing side (x) in each diagram. (Include units and round to 1dp)

a)

i) Use tangent ratio
 $\tan \theta = \frac{O}{A}$

ii) $\tan 32^\circ = \frac{x}{39}$
 $x = 39 \tan 32^\circ$
 $x = 24.4 \text{ km}$

b)

i) Use Cosine Law
 $a^2 = b^2 + c^2 - 2bc \cos A$

$$x^2 = 22^2 + 320^2 - 2(22)(320) \cos 22^\circ$$

$$x^2 = 484 + 102400 - 13054.7$$

$$x^2 = 89829.3$$

$$x = \sqrt{89829.3}$$

$$x = 299.7 \text{ cm}$$

3. For each triangle below:

- Explain which strategy/equation you would use to find the missing angle (θ)
- Find the missing angle (θ) in the diagram below. (Include units and round to 1dp)

a)

i) Cosine ratio
 $\cos \theta = \frac{A}{H}$

ii) $\cos \theta = \frac{19}{39}$
 $\theta = \cos^{-1}(\frac{19}{39})$
 $\theta = 60.8^\circ$

b)

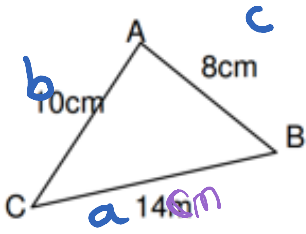
i) Sine law
 $\frac{\sin A}{a} = \frac{\sin B}{b}$

ii) $\frac{\sin \theta}{3.4} = \frac{\sin 43^\circ}{3.7}$
 $3.7 \sin \theta = 3.4 \sin 43^\circ$
 $\sin \theta = \frac{3.4 \sin 43^\circ}{3.7}$
 $\sin \theta = 0.6267$
 $\theta = \sin^{-1}(0.6267)$
 $\theta = 38.8^\circ$

4. a) What does it mean to "solve a triangle"?

Solving a triangle means to find all missing sides and missing angles.

b) Solve the triangle. Summarize your answers in the chart. (Round to 1 dp)



$\angle A = 105^\circ$	$a = 14\text{cm}$
$\angle B = 44.4^\circ$	$b = 10\text{cm}$
$\angle C = 34.1^\circ$	$c = 8\text{cm}$

↳ We know a, b, c. We need A, B, C.
 * Use Cosine Law to find angles

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos A = \frac{10^2 + 8^2 - 14^2}{2(10)(8)}$$

$$\cos A = \frac{100 + 64 - 196}{160}$$

$$\cos A = \frac{-32}{160}$$

$$A = \cos^{-1}\left(\frac{-32}{160}\right) = 101.5^\circ$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos B = \frac{14^2 + 8^2 - 10^2}{2(14)(8)}$$

$$= \frac{160}{224}$$

$$B = \cos^{-1}\left(\frac{160}{224}\right) = 44.4^\circ$$

$$C = 180^\circ - 101.5^\circ - 44.4^\circ = 34.1^\circ$$

5. Angle A is between 0° and 180° . Determine all measures of angle A in each of the following cases:

a) $\sin A = 0.2079$

↳ 2 answers

$$A = \sin^{-1}(0.2079) = 12.0^\circ$$

$$\text{or } A = 180^\circ - 12.0^\circ = 168^\circ$$

b) $\cos A = -0.8191$

↳ obtuse angle

$$A = \cos^{-1}(-0.8191) = 145.0^\circ$$

c) $\tan A = 1.428$

↳ acute angle

$$A = \tan^{-1}(1.428) = 55.0^\circ$$

d) $\tan A = -2.145$

↳ obtuse angle.

$$A = \tan^{-1}(-2.145) = -65^\circ$$

$$A = 180^\circ - 65^\circ = 115^\circ$$

Communication:

6. How do you know when to use SOH CAH TOA? How do you know when to use the Sine Law? How do you know when to use the Cosine Law? Describe in words and given an example.

SOH CAH TOA - use when we have a right angle triangle and we are using sides and angles in our calculation

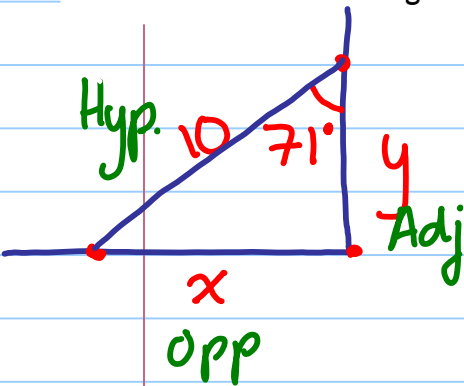
Sine Law - use when the triangle doesn't have a right angle and we know an angle and opposite side

Cosine Law - use when the triangle doesn't have a right angle, and we have 3 sides or 2 sides + contained angle

Word Problems (answer on a separate piece of paper):

Draw and label a diagram for each. Round to 1 dp. Remember to write a "therefore" statement at the end.

7. A ladder 10 feet long is leaning against a wall at a 71° angle. How far from the wall is the foot of the ladder? How high up the wall does the ladder reach?



→ let x represent the distance from the wall

$$\sin \theta = \frac{O}{H}$$

$$\sin 71^\circ = \frac{x}{10}$$

$$x = 10 \sin 71^\circ \\ = 3.3 \text{ ft}$$

∴ The ladder is 3.3 ft from the wall

Let y represent the height of the ladder

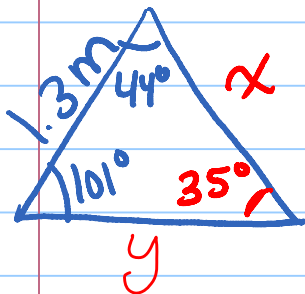
$$\cos \theta = \frac{A}{H}$$

$$\cos 71^\circ = \frac{y}{10}$$

$$y = 10 \cos 71^\circ \\ = 9.5 \text{ ft}$$

∴ The ladder is 9.5 ft high.

8. Billy was making a blueprint of his home, which is triangular in shape. One side of the triangular blueprint is 1.3 meters long. The angles in the triangle at each end of the 1.3m side are 44° and 101° . Determine the lengths of the other two sides of the blueprint.



$$\text{Third angle} = 180^\circ - 44^\circ - 101^\circ \\ = 35^\circ$$

Now we can use the sine law!

$$\frac{x}{\sin 101^\circ} = \frac{1.3}{\sin 35^\circ}$$

$$x \sin 35^\circ = 1.3 \sin 101^\circ$$

$$x = \frac{1.3 \sin 101^\circ}{\sin 35^\circ}$$

$$= 2.2 \text{ m}$$

$$\frac{y}{\sin 44^\circ} = \frac{1.3}{\sin 35^\circ}$$

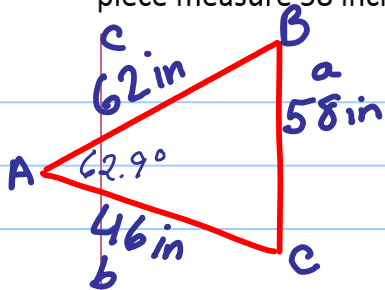
$$y \sin 35^\circ = 1.3 \sin 44^\circ$$

$$y = \frac{1.3 \sin 44^\circ}{\sin 35^\circ}$$

$$= 1.6 \text{ m}$$

∴ The other two sides are 2.2 m and 1.6 m long

9. A machinist is cutting out a large triangular piece of metal to make a part for a crane. The sides of the piece measure 58 inches, 46 inches, and 62 inches. What are the angles between the sides?



$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos A = \frac{(46)^2 + (62)^2 - (58)^2}{2(46)(62)}$$

$$\cos A = \frac{2596}{5704}$$

$$\cos A = 0.4551$$

$$A = \cos^{-1}(0.4551)$$

$$= 62.9^\circ$$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 62.9}{58} = \frac{\sin B}{46}$$

$$58 \sin B = 46 \sin 62.9$$

$$\sin B = \frac{46 \sin 62.9}{58}$$

$$\sin B = 0.7066$$

$$B = \sin^{-1}(0.7066)$$

$$= 44.9^\circ$$

$$C = 180^\circ - A - B$$

$$= 180^\circ - 62.9 - 44.9$$

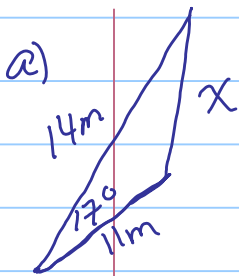
$$= 72.2^\circ$$

\therefore The angles are 44.9° , 62.9° and 72.2°

10. A pole is supported by two guy wires, as shown. One wire is attached to the top of the pole and the other is attached at the middle.

a) Determine the height of the pole.

b) How far from the base of the pole are the wires anchored?



\rightarrow The height is $2x$ because the wire is attached at the midpoint.

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$x^2 = 14^2 + 11^2 - 2(14)(11) \cos 17^\circ$$

$$x^2 = 317 - 294.54$$

$$x^2 = 22.46$$

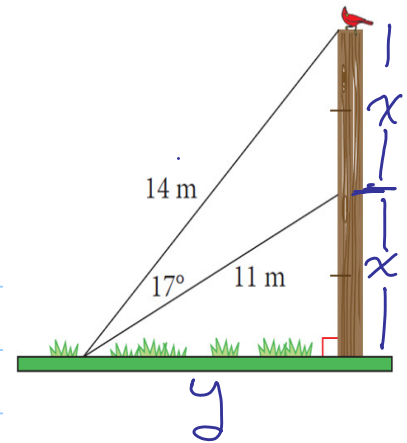
$$x = \sqrt{22.46}$$

$$x = 4.7 \text{ m}$$

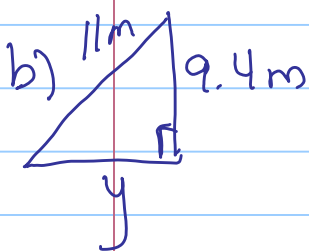
$$\text{Height} = 2x$$

$$= 2(4.7 \text{ m})$$

$$= 9.4 \text{ m}$$



\therefore The height of the pole is 9.4 m.



$$a^2 + b^2 = c^2$$

$$y^2 + 9.4^2 = 11^2$$

$$y^2 = 11^2 - 9.4^2$$

$$y^2 = 32.64$$

$$y = \sqrt{32.64}$$

$$y = 5.7$$

\therefore The wires are 5.7 m from the base of the pole.