

## Zero & Negative Exponents

Complete the following chart. Evaluate each to standard form. Leave as whole numbers or fractions.

Expression to be Simplified	Write in Expanded Form	Using Exponent Laws
$\frac{2^3}{2^1}$	$= \frac{2 \times 2 \times 2}{2}$ $= 4$	$2^{3-1} = 2^2$ $= 4$
$\frac{2^3}{2^2}$	$= \frac{\cancel{2} \times \cancel{2} \times 2}{\cancel{2} \times \cancel{2}} = \frac{4}{2} = 2$	$2^{3-2} = 2^1$ $= 2$
$\frac{2^3}{2^3}$	$= \frac{\cancel{2} \times \cancel{2} \times \cancel{2}}{\cancel{2} \times \cancel{2} \times \cancel{2}} = 1$	$2^{3-3} = 2^0$ $= 1$
$\frac{2^3}{2^4}$	$= \frac{\cancel{2} \times \cancel{2} \times \cancel{2}}{\cancel{2} \times \cancel{2} \times \cancel{2} \times 2} = \frac{1}{2}$	$2^{3-4} = 2^{-1}$ $= \frac{1}{2}$
$\frac{2^3}{2^5}$	$= \frac{\cancel{2} \times \cancel{2} \times \cancel{2}}{\cancel{2} \times \cancel{2} \times \cancel{2} \times 2 \times 2} = \frac{1}{2 \times 2} = \frac{1}{4}$	$2^{3-5} = 2^{-2}$ $= \frac{1}{2^2} = \frac{1}{4}$

**HOW** is the exponent law expression **RELATED TO** the expanded form expression?

**They are the same**

What do you notice about the result of an expression with an **exponent of zero**?

**They are equal to one**

What do you notice about the result of an expression with an **exponent that is negative**?

**The term becomes a fraction  $\frac{1}{\text{base}}$  with an exponent on the base that is positive**

### THE ZERO EXPONENT

Any number (or expression) divided by itself is equal to **one**

Use exponent laws to evaluate each of the following:

a)  $\frac{2^3}{2^3} = 2^{3-3} = 2^0 = 1$

b)  $\frac{3^2}{3^2} = 3^{2-2} = 3^0 = 1$

c)  $\frac{x^4}{x^4} = x^{4-4} = x^0 = 1$

Therefore, for zero exponents: Any **BASE** raised to an **exponent of zero** is equal to **one**

$a^0 = 1$

### EXAMPLES

Evaluate.

$7^0 = 1$

$x^0 = 1$

$3 \times 2^0 = 3 \times 1 = 3$

$x^0 y = 1 \times y = y$

**THE NEGATIVE EXPONENT**

Any BASE raised to a NEGATIVE exponent is equal to the **RECIPROCAL** of the base raised to the same **positive** exponent.

$$\boxed{a^{-m} = \frac{1}{a^m}} \quad \text{and} \quad \boxed{\frac{1}{a^{-m}} = a^m}$$

Use exponent laws to simplify each of the following. Then evaluate to standard form.

$$\begin{aligned} \text{a) } \frac{2^3}{2^4} &= 2^{3-4} = 2^{-1} \\ &= \frac{1}{2} \end{aligned}$$

$$\begin{aligned} \text{b) } \frac{3^2}{3^5} &= 3^{2-5} = 3^{-3} \\ &= \frac{1}{3^3} = \frac{1}{27} \end{aligned}$$

$$\begin{aligned} \text{c) } \frac{4^5}{4^7} &= 4^{5-7} = 4^{-2} \\ &= \frac{1}{4^2} = \frac{1}{16} \end{aligned}$$

**EXAMPLES**

Simplify and evaluate.

$$7^{-1} = \frac{1}{7}$$

$$(-8)^{-2} = \frac{1}{8^2} = \frac{1}{64}$$

$$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

$$(-3)^{-3} = \frac{1}{(-3)^3} = -\frac{1}{27}$$

EXERCISE: Complete the following table.

Exponent Form	$3^2$	$5^{-1}$	$10^0$	$3^{-3}$	$3^{-1}$	$2^{-n}$	$3^{-m}$	$(-1225)^0$
Simplified Form	9	$\frac{1}{5}$	1	$\frac{1}{3^3} = \frac{1}{27}$	$\frac{1}{3}$	$\frac{1}{2^n}$	$\frac{1}{5^m}$	1

**SIMPLIFYING EXPRESSIONS**

The rules for positive exponents also work for zero and negative exponents. Continue to follow the rules for order of operations (BEDMAS) when simplifying & evaluating.

**EXAMPLES**

Simplify and evaluate each of the following:

$$\begin{aligned} 3^3 \times 3^{-5} &= 3^{3+(-5)} \\ &= 3^{3-5} \\ &= 3^{-2} \\ &= \frac{1}{3^2} \\ &= \frac{1}{9} \end{aligned}$$

$$\begin{aligned} \frac{(-2)^2}{(-2)^{-3}} &= (-2)^{2-(-3)} \\ &= (-2)^{2+3} \\ &= (-2)^5 \\ &= -32 \end{aligned}$$

$$\begin{aligned} \left(\frac{3^2}{3^4}\right)^2 &= (3^{2-4})^2 \\ &= (3^{-2})^2 \\ &= 3^{-4} \\ &= \frac{1}{3^4} \\ &= \frac{1}{81} \end{aligned}$$